Worksheet 1: Calculus Review Answers and Solutions

1. *Derivatives*

   (a) See my review of calculus 1 on how to do these or talk to a tutor. Here are the answers:
   a) $y' = 50x^9$ b) $y' = \frac{15\sqrt{x}}{4}$ c) $y' = -6x\sin(3x^2)$ d) $y' = \frac{7}{x} + \frac{6}{x^2}$
   e) $y' = \frac{5}{(5x)^2+1}$ f) $y' = 6\sec(2x+1)\tan(2x+1)$ g) $y' = \frac{2x+4\cos(4x)}{2\sqrt{x^2+\sin(4x)}}$ h) $y' = 2xe^x + x^2e^x$

   (b) These should be quick chain rule problems (Here are the answers):
   i. $\frac{dy}{dx} = 12x(2x^2+4)^2$.
   ii. $\frac{dy}{dx} = 30x(5x^2+3)^2$.
   iii. $\frac{dy}{dx} = 6tx(tx^2+z)^2$.

2. *Integrals*

   (a) See my review of calculus 2 on how to do these or talk to a tutor. Here are the answers:
   a) $\frac{5}{4}x^4 + x + C$ b) $-4\cos(x) + C$ c) $\frac{1}{3}e^{3x} + C$
   d) $4\tan^{-1}(x) + C$ e) $-\cot(x) + C$ f) $\ln(x) - 12\sqrt[3]{x} + C$

   (b) Here are the substitutions I used and the solutions:
   a) Using $u = x^2$ gives $\int xe^{x^2}\,dx = \int \frac{1}{2}e^u\,du = \frac{1}{2}e^u + C = \frac{1}{2}e^{x^2} + C$.
   b) Using $u = x^4$ gives $\int x^3\cos(x^4)\,dx = \int \frac{1}{4}\cos(u)\,du = \frac{1}{4}\sin(u) + C = \frac{1}{4}\sin(x^4) + C$.
   c) Using $u = \ln(x)$ gives $\int \frac{\ln(x)}{x}\,dx = \int u\,du = \frac{1}{2}u^2 + C = \frac{1}{2}(\ln(x))^2 + C$.
   d) Using $u = \tan(x)$ gives $\int \tan^2(x)\sec^2(x)\,dx = \int u^2\,du = \frac{1}{3}u^3 + C = \frac{1}{3}(\tan(x))^3 + C$.

   (c) Here was what I used for u and dv and the solutions:
   a) Using $u = x$ and $dv = e^x dx$ gives $\int xe^x\,dx = xe^x - \int e^x\,dx = xe^x - e^x + C$.
   b) Using $u = x$ and $dv = \sin(x)dx$ gives
   $\int x\sin(x)\,dx = -x\cos(x) + \int \cos(x)\,dx = -x\cos(x) + \sin(x) + C$.
   c) Using $u = \ln(x)$ and $dv = x^2dx$ gives
   $\int x^2\ln(x)\,dx = \frac{1}{3}x^3\ln(x) - \int \frac{1}{3}x^2\,dx = \frac{1}{3}x^3\ln(x) - \frac{1}{9}x^3 + C$.

   (d) Recall that $\sin^2(x) = \frac{1}{2}(1-\cos(2x))$ and $\cos^2(x) = \frac{1}{2}(1+\cos(2x))$ are used for integrals of even powers of sine and cosine.
   i. $\int \sin^2(x)\,dx = \frac{1}{2}\int 1 - \cos(2x)\,dx = \frac{1}{2}x - \frac{1}{4}\sin(2x) + C$
   ii. $\int \cos^4(x)\,dx = \frac{1}{4}\int (1+\cos(2x))^2\,dx = \frac{1}{4}\int 1 + 2\cos(2x) + \cos^2(2x)\,dx$, then using the half-angle identity again on the $\cos^2(2x)$ term gives $\frac{1}{4}\int 1 + 2\cos(2x) + \frac{1}{2} + \frac{1}{2}\cos(4x)\,dx = \frac{1}{4}(\frac{3}{2}x + \sin(2x) + \frac{1}{8}\sin(4x)) + C$

   (e) First we find that $\frac{1}{(x+1)(x+2)} = \frac{1}{x+1} - \frac{1}{x+2}$, then we integrate to get
   $\int \frac{1}{x+1} - \frac{1}{x+2}\,dx = \ln|x+1| - \ln|x+2| + C$.

   (f) We should recognize the necessary substitution is $x = \sec(\theta)$ (also draw the associated triangle) which gives $\int \frac{1}{x^2\sqrt{x^2-1}}\,dx = \int \frac{1}{\sec^2(\theta)\tan(\theta)}\sec(\theta)\tan(\theta)\,d\theta$ which simplifies to
   $\int \frac{1}{\sec(\theta)}\,d\theta = \int \cos(\theta)\,d\theta = \sin(\theta) + C = \frac{\sqrt{x^2-1}}{x} + C$.