Basic Fact Sheet

All proofs on the exam should follow the proper structure and use the precise definitions. You may use, with reference, any of the following theorems. You may also use, without reference, any axiom or proposition from algebra that we have used in the homework. Everything else must be clearly justified from definitions and appropriate logic.

1. **Triangle Inequality Theorem**: $\forall x, y \in \mathbb{R}, |x+y| \leq |x| + |y|$.
2. **AGM Inequality Theorem**: $\forall x, y \in \mathbb{R}$,
   (a) $2xy \leq x^2 + y^2$,
   (b) $xy \leq \left(\frac{x+y}{2}\right)^2$, and
   (c) if $x, y \geq 0$, then $\sqrt{xy} \leq \frac{x+y}{2}$.
3. Basic Logic Facts:

Rule	For Logic	For Sets
de Morgan's Law	$\neg(P \wedge Q) = \neg P \vee \neg Q$	$(A \cap B)^c = A^c \cup B^c$
de Morgan's Law	$\neg(P \vee Q) = \neg P \wedge \neg Q$	$(A \cup B)^c = A^c \cap B^c$
Distributive Laws	$P \vee (Q \wedge R) = (P \vee Q) \wedge (P \vee R)$	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
Distributive Laws	$P \wedge (Q \vee R) = (P \wedge Q) \vee (P \wedge R)$	$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

4. $n \in \mathbb{Z}$ is even $\Leftrightarrow n = 2k$ for some $k \in \mathbb{Z}$.
5. $n \in \mathbb{Z}$ is odd $\Leftrightarrow n = 2k + 1$ for some $k \in \mathbb{Z}$.
6. $y \in f(S) \Leftrightarrow\ y = f(x)$ for some $x \in S$.
7. f is bounded $\Leftrightarrow$ there exists an $M \in \mathbb{R}$ such that $|f(x)| \leq M$ for all $x \in \mathbb{R}$.