Math 300 Summer 2009 Exam 1

Name: ______________________________

Student ID Number: ______________________________

1	24	
2	20	
3	15	
4	21	
Total	80	

- USE YOUR TIME WISELY! START EACH PROBLEM BY OUTLINING THE PROOFS/IDEAS ON THE PAGE. THEN SPEND NO MORE THAN 10-15 MINUTES PER PAGE FILLING IN THE DETAILS (AND THEN MOVE ON TO THE NEXT PAGE).
- I will primarily be giving points for structure and format of proofs. Keep your proofs minimal, but leave the essential structure and steps.
- Put your answers on the exam, there are 4 pages (you may use your own paper for scratch work). If you need more space for your answer use the back of the preceding page and indicate that you have done so.
- If you don't know how to prove something, then you should still let me know everything you know about the problem (that is, give definitions and facts you know about the problem and discuss different techniques to prove the theorem). Also, if you are stuck you should try some examples. Don't leave any question blank, show me what you know!

GOOD LUCK!

1. (a) For real numbers x and y, consider the statement below and answer the questions.
ORIGINAL: "If $x + y \geq 0$, then $x \geq 0$ or $y \geq 0$."

   i. State the contrapositive to the original statement and determine if the original statement is true or false.

   (4 pts) CONTRAPOSITIVE:

   (2 pts) CIRCLE THE TRUTH VALUE OF THE ORIGINAL: TRUE FALSE

   ii. State the converse of the original statement and give a counterexample to the converse.

   (4 pts) CONVERSE:

   (2 pts) COUNTEREXAMPLE TO CONVERSE:

(b) (12 pts) Let A, B, C, and D be sets.
By giving a formal subset proof, show $(A \cap C) \cup (B \cup C)^c \subseteq A \cup B^c \cup D$.

2. (a) Give the negation of the following statement (distribute "not" as far as possible). Then determine if it is true (no proof required):
ORIGINAL: For all $a, b \in \mathbb{Z}$, for all $k \in \mathbb{N}$, if ab^k is even, then a is even or b is even.

(6 pts) NEGATION:

(2 pts) CIRCLE WHICH ONE IS **TRUE**: ORIGINAL NEGATION

(b) (12 points) Consider the four "proofs" below of the theorem:
Theorem: For all $x, y \in \mathbb{Z}$, if x is odd, then $x + 2y$ is odd.

(Pf 1) Assume $x + 2y$ is even. Thus, $x + 2y = 2k$ for some $k \in \mathbb{Z}$. Solving for x gives $x = 2(k - y)$. Hence, x is even. $\square$	(Pf 2) Assume x is odd and $x+2y$ is even. Thus, $x = 2m + 1$ and $x + 2y = 2n$ for some $m, n \in \mathbb{Z}$. Substitution gives $(2m + 1) + 2y = x + 2y = 2n$. Thus, $2m+1+2y = 2n$ which gives $n-m-y = \frac{1}{2}$. Which cannot be true because the sum of integers can't be a fraction. Hence the original theorem is true. $\square$
(Pf 3) Assume $x + 2y$ is odd. Thus, $x+2y = 2k+1$ for some $k \in \mathbb{Z}$. Solving for x gives $x = 2k - 2y + 1 = 2(k - y) + 1$. Hence, x is odd.$\square$	(Pf 4) Assume x is odd. Then $x = 2m + 1$ for some $m \in \mathbb{Z}$. Substitution gives $x + 2y = 2m + 1 + 2y = 2(m + y) + 1$. Thus, $x + 2y$ is odd. $\square$

All intermediate steps in the proofs above are correct. At least one of the "proofs" above is an incorrect proof of the stated theorem.
By filling in the table below, for each proof, tell me if it is a correct proof of the given theorem and, if so, tell me which of the main three proof methods is being used.

	CORRECT PROOF OF GIVEN THEOREM? (YES OR NO)	METHOD OF PROOF (IF CORRECT)
(Pf 1)		
(Pf 2)		
(Pf 3)		
(Pf 4)		

CHECK YOUR TIME!
LAYOUT THE FORMAT OF YOUR PROOFS FIRST, THEN FILL IN AS MANY DETAILS AS YOU CAN. MAKE SURE YOU LEAVE 15+ MINUTES TO WORK ON THE LAST PAGE.

3. (15 pts) Clearly giving a well structure proof and using the precise definitions of even and odd, prove for all x, y in $\mathbb{Z}$, $xy + x$ is even **if and only if** x is even or y is odd.
(Hint: An indirect method might be useful somewhere in your proof.)

4. (a) (13 pts) Using induction, prove that for all $n \in \mathbb{Z}$ with $n \geq 0$, $\displaystyle\sum_{i=0}^{n} \pi^i = \frac{\pi^{n+1} - 1}{\pi - 1}$.

(b) (8 pts) Recall: The AGM(a) states $2xy \leq x^2 + y^2$ and AGM(b) states $xy \leq \left(\frac{x+y}{2}\right)^2$ for all real numbers x and y. Prove for all $x, y \in \mathbb{R}$, if $0 \leq x + y \leq 10$, then $(1 + x)(1 + y) \leq 36$. (Hint: At some point use one of the AGM inequalities.)