Math 300 Summer 2009 Exam 2

Name: ______________________________

1	23	
2	22	
3	15	
4	20	
Total	80	

- USE YOUR TIME WISELY! START EACH PROBLEM BY OUTLINING THE PROOFS/IDEAS ON THE PAGE. THE LAST TWO PAGES ARE PROOFS. SPEND NO MORE THAN 10-20 MINUTES ON THE FIRST TWO PAGES.
- Put your answers on the exam, there are 4 pages (you may use your own paper for scratch work). If you need more space for your answer use the back of the preceding page and indicate that you have done so.
- If you don't know how to prove something, then you should still let me know everything you know about the problem (that is, give definitions and facts you know about the problem and discuss different techniques to prove the theorem). Also, if you are stuck you should try some examples. Don't leave any question blank, give me the opportunity to give you credit for what you know!

GOOD LUCK!

1. (a) Give a counterexample to each following statements:

   i. (4 pts) Every one-to-one function from $\mathbb{R}$ to $\mathbb{R}$ is onto.

   ii. (4 pts) Every unbounded function from $\mathbb{R}$ to $\mathbb{R}$ is monotone.

   (b) (5 pts) Give the coefficient of x^{19} in the expansion of $(x+2)^{21}$.

   (c) (10 pts) Let $g : \mathbb{R} - \{0\} \to \mathbb{R}$ by $g(x) = 5 + \frac{3}{x}$.
   Prove g is injective.

2. (a) (12 pts)

i. Use the Euclidean algorithm to compute $\gcd(208, 84)$.
(You MUST show me the steps of the Euclidean algorithm to get full credit)

ii. Does $208x + 84y = 15$ have a solution for $x, y \in \mathbb{Z}$? If so, find the solution for x and y that is given by the Euclidean algorithm. If not, explain why.

iii. Does $208x + 84y = 20$ have a solution for $x, y \in \mathbb{Z}$? If so, find the solution for x and y that is given by the Euclidean algorithm. If not, explain why.

(b) (10 pts) Prove that if 3 divides a and 3 divides b, then 9 divides $6b + (a + 1)^3 - 1$.

CHECK YOUR TIME! LEAVE 15-20 MINUTES FOR THE LAST PAGE!

3. (15 pts) Define $G_0 = 1$, $G_1 = 1$ and $G_n = 1 + 3G_{n-1} - 2G_{n-2}$ for $n \geq 2$.
   Using strong induction, prove that $G_n = 2^n - n$ for all integers $n \geq 0$.

REMEMBER TO VERY CLEARLY GIVE THE **ORDER AND JUSTIFICATIONS** IN THE PROOFS BELOW.

4. (a) (10 pts) Let $f : \mathbb{R} \to \mathbb{R}$ be a function.
   Define the function $h : \mathbb{R} \to \mathbb{R}$ by $h(x) = 2x - f(x)$ for all $x \in \mathbb{R}$.
   Prove that if f is nonincreasing, then h is increasing.

   (b) (10 pts) Let $f : A \to B$ and $g : B \to C$ be functions and define the function $h : A \to C$ by $h(x) = g(f(x))$ for all $x \in A$.
   Prove that if h is onto and g is one-to-one, then f is onto.
   (Hint: You will use the fact that g is well-defined.)