Math 300 Assignment 3

PROBLEMS: 3.18, 3.26, 3.41, 3.49a, 3.55, 3.56a, 3.57, 3.64, 4.6, 4.8, 4.10, 4.12

PLEASE, PLEASE, PLEASE read the special instructions and hints on the back of this sheet before attempting the problems.

ADDITIONAL REQUIRED PROBLEM

PROBLEM I: Put an 'X' (or 'T') in each entry in the table that is TRUE (No proofs required). Assume the domains and targets for the functions labeled with f, g, h_1 and h_2 are as follows: $f : \mathbb{R} \to \mathbb{R}$, $g : \mathbb{R} - \{0\} \to \mathbb{R}$, $h_1 : \left(-\frac{pi}{2}, \frac{pi}{2}\right) \to \mathbb{R}$, and $h_2 : \mathbb{R} - \{a : a = \frac{k\pi}{2}, k \in \mathbb{Z}\} \to \mathbb{R}$.
Note that the domains and targets given will effect whether the function is injective, surjective, or monotone.

FUNCTION	INJECTIVE?	SURJECTIVE?	BIJECTIVE?	MONOTONE?
$f(x) = 5$				
$f(x) = 3x + 1$				
$f(x) = x^2$				
$f(x) = x^3$				
$f(x) = x^3 - x$				
$f(x) = e^x$				
$f(x) = \sin(x)$				
$f(x) = \arctan(x)$				
$g(x) = 1/x$				
$h_1(x) = \tan(x)$				
$h_2(x) = \tan(x)$				

I hope that this table helps you better understand these terms and it will help you come up with counterexamples on the rest of the homework (especially 4.12). Please ask me if you are having trouble filling in this table. We will discuss chapter at the end of class on Monday and during lecture on Wednesday.

The problems above are DUE FRIDAY, JULY 22th at lecture or during office hours.

HOMEWORK NOTES/HINTS

- PROBLEM 3.41: Part (a) should be quick (at most three lines). That is, don't use induction just plug in particular values of x and y to deduce the result (which is okay because the result is true for all x and y). But induction must be used on part (b).

- PROBLEMS 3.55, 3.56a, and 3.57: All of these problems use second order recurrences, so you will need to show the first two cases in your base step so that you can use the recurrence formula in the inductive step. Then note that you are technically using strong induction when you are using an inductive hypothesis that involves more than just the previous step.

- PROBLEM 3.64: Here is the layout of the proof: Give a proof by contradiction. That is, start by assuming there is a statement $P(n)$ such that both the hypotheses of induction are true but the conclusion is false (namely $P(n)$ is not true for some positive integers n). Let S be the set of all positive integers such that $P(n)$ is false. Then use the well ordering principle to say something about this set and find a contradiction with one of the hypotheses of induction. (I've written almost the whole proof here, you just make it look nice and finish it correctly).

  Note, this proves that if the well-ordering principle is true, then the principal of mathematical induction is true (you can do an argument in reverse as well, so the two theorems are logically equivalent).

- PROBLEM 4.10: Prove that $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = ax + b$ with $a \neq 0$ is injective and surjective (don't give the proof for $g(x)$ because it would be identical). Then explain why $h(x) = g(f(x)) - f(g(x))$ would not be injective and would not be surjective from $\mathbb{R}$ to $\mathbb{R}$.

- PROBLEM 4.12: Only part (d) is true (use a contrapositive or contradiction proof). For the rest find counterexamples. Note: part (c) is challenging in the sense that you may want to try to draw a picture first, then see if you can come up with a function (think discontinuous functions).

- If you finish the homework early or if you are looking for some extra practice try the following problems:

  CHALLENGE PROBLEMS: 3.38, 3.39, 3.65

  These are not due, but I will award at least 1 point of extra credit per challenge problem correctly completed.