NAME: SOL'N

Test Prep 4

If you finish this page, try the problem on the back (for an extra point). This is open book, you can use any notes you have. You have 10 minutes.

1. Give the Laplace Transform for $f(t) = 7 + e^{-2t}\cos(5t) + u_5(t)\sin(4(t-5))$.

$$\frac{7}{s} + \frac{s+2}{(s+2)^2+25} + e^{-5s}\mathcal{L}\{\sin(4t)\}$$

$$\boxed{\frac{7}{s} + \frac{(s+2)}{(s+2)^2+25} + e^{-5s}\frac{4}{s^2+16}}$$

2. Give the inverse Laplace Transform for $F(s) = \dfrac{s+20}{(s+2)^2+16} + e^{-2s}\dfrac{1}{s-6}$.

$$\mathcal{L}^{-1}\left\{\frac{(s+2)}{(s+2)^2+16}\right\} + \mathcal{L}^{-1}\left\{\frac{18}{(s+2)^2+16}\right\} + u_2(t)\underbrace{\mathcal{L}^{-1}\left\{\frac{1}{s-6}\right\}}_{e^{6u}}(t+2)$$

$$\boxed{e^{-2t}\cos(4t) + \frac{18}{4}e^{-2t}\sin(4t) + u_2(t)\,e^{6(t-2)}}$$

$\frac{9}{2}$

3. Consider $h(t) = 5u_3(t) + u_6(t)(t-6) + u_{10}(t)$. If we write $h(t) = \begin{cases} A, & t<3; \\ B, & 3\le t<6; \\ C, & 6\le t<10; \\ D, & t\ge 10, \end{cases}$, then what are the numbers/functions (possibly involving t) that should be in place of A, B, C, and D?

$A = 0$ ← $5\cdot 0 + 0\cdot(t-6) + 0$

$B = 5$ ← $5\cdot 1 + 0\cdot(t-6) + 0$

$C = t-1$ ← $5\cdot 1 + 1\cdot(t-6) + 0$

$D = t$ ← $5\cdot 1 + 1\cdot(t-6) + 1$

4. Consider $y'' - y = g(t) = \begin{cases} 6 & , t < 3; \\ 10 & , t \geq 3. \end{cases}$ with $y(0) = 0$ and $y'(0) = 0$.

Take the LaPlace transform of both sides and simplify to get $\mathcal{L}\{y\} = ??$. (*i.e.* You don't have to completely solve for y, just simplify and tell me what goes in place of the questions marks)

$$\left(s^2\mathcal{L}\{y\} - s\underbrace{y(0)}_{0} - \underbrace{y'(0)}_{0}\right) - \mathcal{L}\{y\} = \mathcal{L}\{6 + 4u_3(t)\}$$

$$(s^2-1)\mathcal{L}\{y\} = \frac{6}{s} + 4e^{-3s}\frac{1}{s}$$

$$\mathcal{L}\{y\} = \frac{6}{s(s^2-1)} + e^{-3s}\frac{4}{s(s^2-1)} = (6+4e^{-3s})\frac{1}{s(s^2-1)}$$

GOING FURTHER

$$\frac{1}{s(s+1)(s-1)} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s-1} \Rightarrow$$

$$A = \frac{1}{(0+1)(0-1)} = -1$$

$$B = \frac{1}{(-1)(-1-1)} = \frac{1}{2}$$

$$C = \frac{1}{(1)(1+1)} = \frac{1}{2}$$

$$\mathcal{L}^{-1}\left\{\frac{-1}{s} + \frac{1/2}{s+1} + \frac{1/2}{s-1}\right\} = -t + \frac{1}{2}e^{-t} + \frac{1}{2}e^{t}$$

aside: $\cosh(t)$

THUS,

$$y = (-6t + 3e^{-t} + 3e^{t}) + u_3(t)\left(\underbrace{-4(t-3)}_{-4t+12} + \underbrace{2e^{-(t-3)}}_{+2e^3e^{-t}} + \underbrace{2e^{(t-3)}}_{+2e^{-3}e^{t}}\right)$$

$$y = \begin{cases} -6t + 3e^{-t} + 3e^{t}, & t < 3; \\ -10t + 12 + (3+2e^3)e^{-t} + (3+2e^{-3})e^{t}, & t \geq 3. \end{cases}$$

5. Any questions for your instructor? Let me know.