3.1, 3.3, 3.4: Homogeneous Constant Coefficient 2nd Order

Given $ay'' + by' + cy = 0$, $y(t_0) = y_0$ and $y'(t_0) = y_0'$.

Step 1: Write the characteristic equation $ar^2 + br + c = 0$ and find the roots $r = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Step 2: Write your answer in the appropriate form:

1. If $b^2 - 4ac > 0$, then there are two real roots r_1 and r_2 and the general solution is
$$y(t) = c_1 e^{r_1 t} + c_2 e^{r_2 t}.$$
2. If $b^2 - 4ac = 0$, then there is one real root r and the general solution is
$$y(t) = c_1 e^{rt} + c_2 t e^{rt}.$$
3. If $b^2 - 4ac < 0$, then there are two complex roots $r = \lambda \pm \omega i$ and the general solution is
$$y(t) = e^{\lambda t} \left(c_1 \cos(\omega t) + c_2 \sin(\omega t) \right).$$

Step 3: Use initial conditions

1. Find $y'(t)$.
2. Plug in $y(t_0) = y_0$.
3. Plug in $y'(t_0) = y_0'$.
4. Combine and solve for c_1 and c_2.

Several quick examples (answers on back):

1. Solve $y'' + 2y' + y = 0$.
2. Solve $y'' - 10y' + 24y = 0$.
3. Solve $y'' + 5y = 0$.
4. Solve $y'' - 3y' = 0$.
5. Solve $y'' + 12y' + 36y = 0$.
6. Solve $y'' + y' + y = 0$.

Several quick examples:

1. $r^2 + 2r + 1 = (r+1)^2 = 0$:
   $y(t) = c_1 e^{-t} + c_2 t e^{-t}$.

2. $r^2 - 10r + 24 = (r-6)(r-4) = 0$:
   $y(t) = c_1 e^{6t} + c_2 e^{4t}$.

3. $r^2 + 5 = 0$, $r = \pm\sqrt{5}i$:
   $y(t) = c_1 \cos(\sqrt{5}t) + c_2 \sin(\sqrt{5}t)$.

4. $r^2 - 3r = r(r-3) = 0$:
   $y(t) = c_1 + c_2 e^{3t}$

5. $r^2 + 12r + 36 = (r+6)^2 = 0$:
   $y(t) = c_1 e^{-6t} + c_2 t e^{-6t}$

6. $r^2 + r + 1 = 0$, $r = \frac{-1}{2} \pm \frac{\sqrt{3}}{2} i$:
   $y(t) = e^{-t/2} \left(c_1 \cos(\sqrt{3}t/2) + c_2 \sin(\sqrt{3}t/2) \right)$