Skills Review: Trigonometry and Waves

The following review discusses some trigonometry, specifically facts related to waves.

Introduction and Basic Facts:
Consider functions of the form $y(t) = A\cos(\omega t - \delta)$. Our book likes to express waves in this standard form. The graph of this function looks like a wave which is oscillating about the t-axis. Here are several important facts about this wave:

- A = 'the amplitude' = 'the distance from the middle of the wave to the highest point'
- ω = 'angular frequency' = 'how many radians between $t = 0$ and $t = 1$'.
- $\omega = 2\pi f$, where f = 'the frequency' = 'the number of full waves between $t = 0$ and $t = 1$'
- $\omega = \frac{2\pi}{T}$ or, in other words,
  $T = \frac{1}{f}$ = 'the period (or wavelength)' = 'distance on the t-axis between peaks'
- δ = 'phase (or phase shift)' = 'the *starting* angle that corresponds to $t = 0$'.

A full example with a picture is on the next page.

Converting into Standard Form:
In this class, we often will have solutions involving expressions of the form $y = A\cos(\mu t) + B\sin(\mu t)$. In order to write this in the form above, you need the trig identity:

$$y = R\cos(\omega t - \delta) = R\cos(\delta)\cos(\omega t) + R\sin(\delta)\sin(\omega t).$$

Setting this equal to $y = A\cos(\mu t) + B\sin(\mu t)$, we conclude that $\omega = \mu$, $A = R\cos(\delta)$, and $B = R\sin(\delta)$. And from these relationships we can conclude that $R^2 = A^2 + B^2$. Therefore, we get

$$R = \sqrt{A^2 + B^2}\ ,\ A = R\cos(\delta)\ ,\ \text{ and }\ B = R\sin(\delta)$$

Example: Consider $y = \frac{7\sqrt{3}}{2}\cos(12\pi t) + \frac{7}{2}\sin(12\pi t)$.

To write in the standard form above, we want $R = \sqrt{\left(\frac{7\sqrt{3}}{2}\right)^2 + \left(\frac{7}{2}\right)^2} = \sqrt{\frac{49(3+1)}{4}} = 7$.

We also want $\frac{7\sqrt{3}}{2} = 7\cos(\delta)$ and $\frac{7}{2} = 7\sin(\delta)$ which gives $\delta = \frac{\pi}{6}$.
Therefore, we get $y = 7\cos\left(12\pi t - \frac{\pi}{6}\right)$. A graph of this function is on the next page.

For example: Consider $y(t) = 7\cos\left(12\pi t - \frac{\pi}{6}\right)$. Let's say t is in minutes just to give some units. A picture is provided below.

1. $A = 7$ is the amplitude. So this wave oscillates between $y = -7$ and $y = 7$.

2. $\omega = 12\pi$ radians per minute. In other words, every minute we will add all radians from 0 to 12π.

3. $f = \frac{\omega}{2\pi} = 6$ waves per minute. In other words, every minute there will be 6 full waves (a full wave is peak-to-peak, or valley-to-valley).

4. $T = \frac{1}{f} = \frac{1}{6}$ minutes per wave. In other words, it take $\frac{1}{6}$ minute (*i.e.* 10 seconds) to complete complete one full wave.

5. $\delta = \frac{\pi}{6}$ is the 'starting angle'. In other words, when $t = 0$, the wave starts at $y(0) = 7\cos(-\frac{\pi}{6}) = 7\sqrt{3}/2$. From here the wave will go up (because this is what the Cosine wave does after $-\pi/6$) and it will complete one wave in 1/6 minute (10 seconds). After these 10 seconds, it will be back to the value of $y(1/6) = 7\sqrt{3}/2$ and the wave will continue in this way.