Math 307 - Spring 2016
Exam 1
April 20, 2016

Name: ______________________________

Student ID Number: ______________________________

- There are 5 pages of questions. Make sure your exam contains all these questions.
- You are allowed to use a scientific calculator (**no graphing calculators and no calculators that have calculus capabilities**) and one **hand-written** 8.5 by 11 inch page of notes.
- You must show your work on all problems. The correct answer with no supporting work may result in no credit. **Put a box around your FINAL ANSWER for each problem and cross out any work that you don't want to be graded.** Give exact answers wherever possible.
- If you need more room, use the backs of the pages and indicate to the grader that you have done so.
- Raise your hand if you have a question.
- There may be multiple versions of the exam so if you copy off a neighbor and put down the answers from another version we will know you cheated. Any student found engaging in academic misconduct will receive a score of 0 on this exam. All suspicious behavior will be reported to the student misconduct board. In such an instance, you will meet in front of a board of professors to explain your actions. I have turned in many cases of suspicious work in the past. I will not hesitate to report suspicious work. This is your warning!
  Avoid suspicion of cheating by keeping your eyes on your paper and clearly showing your work on each problem!
- You have 50 minutes to complete the exam. Budget your time wisely.
  SPEND NO MORE THAN 10 MINUTES PER PAGE!

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Total	50	

GOOD LUCK!

1. (11 pts)

(a) Find the general explicit solution to $\dfrac{dy}{dt} + \dfrac{2}{t} y = t$.

(b) Consider $\dfrac{dy}{dt} = (1 + 4e^t)(y - 3)^2$. (Put final solutions in the boxes provided)

i. Find the explicit solution with $y(0) = 4$. Solution:

ii. Find the explicit solution with $y(5) = 3$. Solution:

2. (11 pts)

(a) Find an implicit solution to $(5xe^y + 4y^3)\dfrac{dy}{dx} = -5e^y$ with $y(1) = 0$.

(b) Consider $2y\dfrac{dy}{dx} = 3e^x + y^2$ with $y(0) = 2$.
Use $u = y^2$ to transform the differential equation into one involving x, u, and $\frac{du}{dx}$.
Solve the new equation and give the explicit solution to the original initial value problem.

3. (8 pts) Dr. Loveless filled his bathtub, pulled the plugged and used a stopwatch to record how long it took it to empty. Using the model from one of your homework questions and replacing the constants with measurements from the bathtub, we get the differential equation

$$\frac{dh}{dt} = -0.002\sqrt{19.6h},$$

where $h(t)$ is height in meters of the water in the bathtub after t seconds.
The initial height of the water is $h(0) = 0.4$ meters.

(a) Find the explicit solution to the differential equation.

(b) According to this model, how long does it take for Dr. Loveless' bathtub to empty?

4. (8 pts) Water drips into a cone shaped hole in the ground at a constant rate and also evaporates. Using the model from homework and some estimates for constants, we get the differential equation:

$$\frac{dV}{dt} = 10 - 0.5V^{2/3},$$

where $V(t)$ is the volume of water in the puddle in cubic inches after t minutes.
Assume the puddle is initially empty (so $V(0) = 0$).

(a) Use Euler's method with step size $h = 0.5$ to estimate the volume of the water in the puddle in $t = 1$ minute. Round your final answer to 3 digits after the decimal.

(b) Without solving, determine the value of $\lim_{t\to\infty} V(t)$. Explain your answer

5. (12 pts) These are all short answer questions!

(a) A spherical snowball melts (*loses* volume) at a rate proportional to its surface area with a proportionality constant that has magnitude 0.1 in/min. Give the differential equation for the volume that only involves the variables t and V. (DO NOT SOLVE)
For reference: For a sphere, Volume $= V = \frac{4}{3}\pi r^3$, Surface Area $= S = 4\pi r^2$.

(b) Historical data suggests that the population in a certain town grows at a rate proportional to the population size with proportionality constant $k = 0.03$. Due to an economic downturn that starts today, jobs become scarce and people start to leave at a constant rate of 300 people per year.
Let $y(t)$ be the number of people in the town t years after the economic downturn started.

i. Write down the differential equation that models the population.
(Big Hint: This is just like a bank account problem; Population is growing at a 3% annual rate and 300 people are 'withdrawing' from the town each year)

ii. Find all equilibrium solutions and classify them as stable, unstable or semistable.

iii. Assume the town started with 4000 people. What will happen to the population size in this town as t increases? Explain your answer.