1. Solve the IVP

$$\begin{aligned} y'' - 4y' + 4y &= e^{2t}\cos(3t) + 4 \\ y(0) &= 0 \\ y'(0) &= 0. \end{aligned}$$

2. An object weighing 96 lb is attached to a spring, stretching it 2 feet. Assume there is no damping, and that an external force $F(t) = 3\sin(4t) - \cos(4t)$ is applied to the object. At time $t = 0$, you push the object 3 feet upward from equilibrium position and give it an initial velocity of 1 ft/s downward. Find the position of the object at time t.

(Recall $g = 32\text{ft/s}^2$).

3. A 10kg rock is attached to a spring, stretching it 2 meters.

(a) For this part only, assume there is no damping, and no external force. If at $t = 0$ the spring is stretched downward by 2m and the rock is released with initial velocity 7 m/s upward, find the period, amplitude, and phase of the motion (Your answer for the phase may involve a trigonometric function).

(b) Now assume there is damping, and that the magnitude of the damping force is 12 N when the object is traveling at 2 m/s. Find the quasi-period of the motion.

(c) How large does the damping force need to be for the system to be critically damped?

4. Given that $y_1(t) = 1/t$ is a solution to the following equation, find another solution:

$$t^2y'' + 3ty' + y = 0, \quad t > 0.$$