Chapter 3: 2nd Order Equations Introduction

Some Introductory Facts About Second Order Equations:

1. A **second order** ordinary differential equation (ODE) is any equation that can be written in form: $y'' = f(t, y, y')$. The terms **second order** indicate that the 2nd derivative appears in the differential equation.

2. A second order ODE is **linear** if it can be written in the form: $y'' + p(t)y' + q(t)y = g(t)$. If the equation cannot be written in this form, we say it is **nonlinear**.

3. *Comment about nonlinear second order equations*: For first order equation, we saw that all linear equation could be solved using integrating factors, but nonlinear differential equations required specialized techniques (separation and exact) that don't always work. For second order systems, things only get worst. In this course, we are NOT going to study nonlinear second order differential equations.

4. *Initial Conditions*: In order to solve second order differential equations you typically are given **TWO initial conditions**. Sometimes you might be given two values of the initial function $y(t_0) = y_0$ and $y(t_1) = y_1$, but typically you're given a value of the function, $y(t_0) = y_0$, and a value of the first derivative, $y'(t_0) = y_0'$. Thus, we will be concerned with solving differential equations of the form:

$$y'' + p(t)y' + q(t)y = g(t), \quad y(t_0) = y_0, \quad y'(t_0) = y_0'.$$

5. A 2nd order linear differential equation is said to have **constant coefficients** if it can be written in the form $ay'' + by' + cy = g(t)$, where a, b and c are constants (with $a \neq 0$).

6. A **homogeneous** equation is one where $g(t) = 0$ for all t. Otherwise, we say the equation is **non-homogeneous**.

7. Here is a break down of chapter 3 (we won't do everything from all of these sections):
   (a) Section 3.1: Intro. The **characteristic equation** is introduced. About $ay'' + by' + cy = 0$.
   (b) Section 3.2: Theorems and Facts about 2nd order systems. Wronskian introduced.
   (c) Section 3.3: Complex characteristic roots (still talking about $ay'' + by' + cy = 0$).
   (d) Section 3.4: Repeated characteristic roots (still only talking about $ay'' + by' + cy = 0$.)
   (e) Section 3.5: Undetermined Coefficients: (now we talk about $ay'' + by' + cy = g(t)$.)
   (f) Section 3.6: Variation of Parameters: A method for $y'' + p(t)y' + q(t)y = g(t)$.
   (g) Section 3.7: Applications (Vibrations/Oscillations)
   (h) Section 3.8: More applications (Forced Vibrations)

8. *Comment*: Most of our second order methods will be about recognizing the form of the equation. You might classify some of our methods as "educated guess and check". Meaning that we 'guess' at the form of the answer and hope that we can pick parameters and coefficients in order to make our function satisfy the differential equation. Once we build an archive of good 'guesses', we won't really be guessing any more, we will just be recognizing the solution based on the form.