Chapter 6: Laplace Transform Introduction

The Problem:
In this section we primarily solving linear constant coefficient with forcing of the form

$$ay'' + by' + cy = g(t).$$

These are the same problems we considered in 3.5, 3.6, and 3.8. However, in many engineering applications, the function $g(t)$ is some sort of discontinuous step or impulse function (the force turns on and off). In these cases, the methods of 3.5, 3.6, and 3.8 can be awkward. The method of Laplace Tranforms is a general method that solve these problems in a systematic (algebraic) way.

Laplace Transform:
For a given function $f(t)$, we define the **Laplace transform**, $\mathcal{L}f(t)$, by

$$F(s) = \mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)\, dt.$$

The idea:

1. Take the Laplace transform of the differential equation to turn it into an algebraic equation involving s.
2. Solve the algebraic problem.
3. Invert the transform to get the solution.

This particular transform works well for linear differential equations because the solutions often involve exponential functions.

Here is a break down of chapter 6 (we won't do everything from all of these sections):

1. Section 6.1: Intro. The Laplace transform is introduced and we build a collection of known evaluations. You'll need to remember integration by parts.
2. Section 6.2: We learn how to solve $ay'' + by' + cy = g(t)$ with a Laplace transform. You'll need to know partial fractions.
3. Section 6.3: Discontinuous forcing functions are introduced (the unit step function).
4. Section 6.4: We learn to solve $ay'' + by' + cy = g(t)$ when $g(t)$ is not continuous.
5. Section 6.5: The unit impulse function.