Inverse Laplace Transform Practice Problems

(Answers on the last page)

(A) Continuous Examples (no step functions): Compute the inverse Laplace transform of the given function.

- The same table can be used to find the inverse Laplace transforms. But it is useful to rewrite some of the results in our table to a more user friendly form. In particular:
  $\mathcal{L}^{-1}\{\frac{1}{s^2+b^2}\} = \frac{1}{b}\sin(bt)$.
  $\mathcal{L}^{-1}\{\frac{1}{(s+a)^2+b^2}\} = \frac{1}{b}e^{at}\sin(bt)$.
  $\mathcal{L}^{-1}\{\frac{1}{s^n}\} = \frac{1}{(n-1)!}t^{n-1}$.
  $\mathcal{L}^{-1}\{\frac{1}{(s-a)^n}\} = \frac{1}{(n-1)!}t^{n-1}e^{at}$.

In the problems below, you are given an expression that has already been obtained by using partial fractions. In a full problem, you would have to do partial fractions to get to this form.

1. $\frac{4}{s-2} - \frac{3}{s+5}$
2. $\frac{s+5}{s^2+9} = \frac{s}{s^2+9} + \frac{5}{s^2+9}$
3. $\frac{5(s+2)-4}{(s+2)^2+9} = \frac{5(s+2)}{(s+2)^2+9} - \frac{4}{(s+2)^2+9}$
4. $\frac{4}{s} - \frac{1}{s^2} + \frac{5}{s^3} + \frac{2}{s^4}$
5. $\frac{10}{(s-5)^2} + \frac{2}{(s-5)^3}$
6. $\frac{1}{s^2+6s+13}$ (start by completing the square)

(B) Discontinuous Examples (step functions): Compute the Laplace transform of the given function.

- Use $\mathcal{L}^{-1}\{e^{-cs}F(s)\} = u_c(t)\mathcal{L}\{F(s)\}(t-c)$.
  Thus, you 'pull out' e^{-cs} and write $u_c(t)$ out in front.
  You then find the Laplace transform of $F(s)$ in the table, but you replace ever 't' with '$t-c$'.

Practice problems:

1. $\frac{e^{-2s}}{s} + \frac{6e^{-3s}}{s}$
2. $e^{-3s}\left(\frac{1}{s^2} + \frac{5}{s^3}\right)$
3. $6 + \frac{e^{-s}}{s^2+4}$
4. $\frac{e^{-5s}(s+1)}{(s+1)^2+16}$
5. $\frac{4e^{-2s}}{s-3} + \frac{e^{-5s}}{s+9}$
6. $\frac{e^{-10s}}{(s-3)^2}$
7. $\frac{e^{-7s}}{s} + \frac{e^{-11s}}{(s-2)^3}$

(A) Answers to continuous examples:

1. $\mathcal{L}^{-1}\{\frac{4}{s-2} - \frac{3}{s+5}\} = 4e^{2t} - 3e^{-5t}$

2. $\mathcal{L}^{-1}\{\frac{s}{s^2+9} + \frac{5}{s^2+9}\} = \cos(3t) + \frac{5}{3}\sin(3t)$

3. $\mathcal{L}^{-1}\{\frac{8(s+2)}{(s+2)^2+25} - \frac{4}{(s+2)^2+25}\} = 8e^{-2t}\cos(5t) - \frac{4}{5}e^{-2t}\sin(5t)$

4. $\mathcal{L}^{-1}\{\frac{4}{s} - \frac{1}{s^2} + \frac{5}{s^3} + \frac{2}{s^4}\} = 4 - t + \frac{5}{2!}t^2 + \frac{2}{3!}t^3$

5. $\mathcal{L}^{-1}\{\frac{10}{(s-5)^2} + \frac{2}{(s-5)^3}\} = 10te^{5t} + \frac{2}{2!}t^2e^{5t} = 10te^{5t} + te^{5t}$

6. $\mathcal{L}^{-1}\{\frac{1}{(s+3)^2+4}\} = \frac{1}{2}e^{-3t}\sin(2t)$

(B) Answers to discontinuous examples:

1. $\mathcal{L}^{-1}\{\frac{e^{-2s}}{s}\} + \mathcal{L}^{-1}\{\frac{6e^{-3s}}{s}\} = u_2(t) + 6u_3(t)$.

2. $\mathcal{L}^{-1}\{e^{-3s}\left(\frac{1}{s^2} + \frac{5}{s^3}\right)\} = u_3(t)\mathcal{L}^{-1}\{\frac{1}{s^2} + \frac{5}{s^3}\}(t-3) = u_3(t)\left((t-3) + \frac{5}{2!}(t-3)^2\right)$.

3. $\mathcal{L}^{-1}\{6 + \frac{e^{-s}}{s^2+4}\} = \frac{6}{s} + u_1(t)\mathcal{L}^{-1}\{\frac{1}{s^2+4}\}(t-1) = \frac{6}{s} + u_1(t)\sin(2(t-1))$.

4. $\mathcal{L}^{-1}\{\frac{e^{-5s}(s+1)}{(s+1)^2+16}\} = u_5(t)\mathcal{L}^{-1}\{\frac{(s+1)}{(s+1)^2+16}\}(t-5) = u_5(t)e^{-(t-5)}\cos(4(t-5))$.

5. $\mathcal{L}^{-1}\{\frac{4e^{-2s}}{s-3}\} + \mathcal{L}^{-1}\{\frac{e^{-5s}}{s+9}\} = 4u_2(t)\mathcal{L}^{-1}\{\frac{1}{s-3}\}(t-2) + u_5(t)\mathcal{L}^{-1}\{\frac{1}{s+9}\}(t-5) = 4u_2(t)e^{3(t-2)} + u_5(t)e^{-9(t-5)}$.

6. $\mathcal{L}^{-1}\{\frac{e^{-10s}}{(s-3)^2}\} = u_{10}(t)\mathcal{L}^{-1}\{\frac{1}{(s-3)^2}\}(t-10) = u_{10}(t)(t-10)e^{3(t-10)}$.

7. $\mathcal{L}^{-1}\{\frac{e^{-7s}}{s}\} + \mathcal{L}^{-1}\{\frac{e^{-11s}}{(s-2)^3}\} = u_7(t) + u_{11}(t)\frac{1}{2!}(t-11)^2e^{2(t-11)}$.