Laplace Transform Practice Problems

(Answers on the last page)

(A) Continuous Examples (no step functions): Compute the Laplace transform of the given function.

1. $e^{4t} + 5$
2. $\cos(2t) + 7\sin(2t)$
3. $e^{-2t}\cos(3t) + 5e^{-2t}\sin(3t)$
4. $10 + 5t + t^2 - 4t^3$
5. $(t^2 + 4t + 2)e^{3t}$
6. $6e^{5t}\cos(2t) - e^{7t}$

(B) Discontinuous Examples (step functions): Compute the Laplace transform of the given function.

- First, rewrite in terms of step functions!
  To do this at each step you 'add the jump'. That is, if the formula changes from $g_1(t)$ to $g_2(t)$ at $t = c$, then you will have a term of the form $u_c(t)(g_2(t) - g_1(t))$ in the function.
- Second, use $\mathcal{L}\{u_c(t)f(t-c)\} = e^{-cs}\mathcal{L}\{f(t)\}$.
  Another way to write this is $\mathcal{L}\{u_c(t)f(t)\} = e^{-cs}\mathcal{L}\{f(t+c)\}$.
  Thus, you 'pull out' $u_c(t)$ and write e^{-cs} out in front. At the same time you replace 't' with '$t + c$' and find the Laplace function of the new expression.

Practice problems:

1. $f(t) = \begin{cases} 0 & ,0 \le t < 6; \\ 3 & ,t \ge 6. \end{cases}$
2. $g(t) = \begin{cases} 3 & ,0 \le t < 5; \\ 10 & ,5 \le t \le 8 \\ 0 & ,t \ge 8. \end{cases}$
3. $h(t) = \begin{cases} 0 & ,0 \le t < 3; \\ 6\sin(t-3) & ,t \ge 3. \end{cases}$
4. $j(t) = \begin{cases} 4 & ,0 \le t < 2; \\ 4 + 5(t-2)e^{t-2} & ,t \ge 2. \end{cases}$
5. $u(t) = \begin{cases} 0 & ,0 \le t < 7; \\ (t-7)^3 & ,t \ge 7. \end{cases}$
6. $v(t) = \begin{cases} 5 & ,0 \le t < 1; \\ t & ,t \ge 1. \end{cases}$
7. $w(t) = \begin{cases} 2 & ,0 \le t < 4; \\ t^2 & ,t \ge 4. \end{cases}$

(A) Answers to continuous examples:

1. $\mathcal{L}\{e^{4t}+5\} = \frac{1}{s-4} + \frac{5}{s}$
2. $\mathcal{L}\{\cos(2t) + 7\sin(2t)\} = \frac{s}{s^2+4} + \frac{7\cdot 2}{s^2+4} = \frac{s+14}{s^2+4}$
3. $\mathcal{L}\{e^{-2t}\cos(3t) + 5e^{-2t}\sin(3t)\} = \frac{(s+2)}{(s+2)^2+9} + \frac{5\cdot 3}{(s+2)^2+9} = \frac{(s+2)+15}{(s+2)^2+9}$
4. $\mathcal{L}\{10 + 5t + t^2 - 4t^3\} = \frac{10}{s} + \frac{5}{s^2} + \frac{2!}{s^3} - \frac{4\cdot 3!}{s^4} = \frac{10}{s} + \frac{5}{s^2} + \frac{2}{s^3} - \frac{24}{s^4}$
5. $\mathcal{L}\{(t^2+4t+2)e^{3t}\} = \mathcal{L}\{t^2e^{3t} + 4te^{3t} + 2e^{3t}\} = \frac{2}{(s-3)^3} + \frac{4}{(s-3)^2} + \frac{2}{s-3}$
6. $\mathcal{L}\{6e^{5t}\cos(2t) - e^{7t}\} = \frac{6(s-5)}{(s-5)^2+4} - \frac{1}{s-7}$

(B) Answers to discontinuous examples:

1. Write $f(t) = 3u_6(t)$.
   $\mathcal{L}\{3u_6(t)\} = e^{-6s}\mathcal{L}\{3\} = \frac{3e^{-6s}}{s}$.
2. Write $g(t) = 3 + (10-3)u_5(t) + (0-10)u_8(t) = 3 + 7u_5(t) - 10u_8(t)$.
   $\mathcal{L}\{3 + 7u_5(t) - 10u_8(t)\} = \frac{3}{s} + \frac{7e^{-5s}}{s} - \frac{10e^{-8s}}{s}$.
3. Write $h(t) = 6u_3(t)\sin(t-3)$.
   $\mathcal{L}\{6u_3(t)\sin(t-3)\} = 6e^{-3s}\mathcal{L}\{\sin(t)\} = \frac{6e^{-3s}}{s^2+1}$.
4. Write $j(t) = 4 + 5u_2(t)(t-2)e^{t-2}$.
   $\mathcal{L}\{4 + 5u_2(t)(t-2)e^{t-2}\} = \frac{4}{s} + 5\mathcal{L}\{u_2(t)(t-2)e^{t-2}\} = \frac{4}{s} + 5e^{-2s}\mathcal{L}\{te^t\} = \frac{4}{s} + \frac{5e^{-2s}}{(s-1)^2}$
5. Write $u(t) = u_7(t)(t-7)^3$.
   $\mathcal{L}\{u_7(t)(t-7)^3\} = e^{-7s}\mathcal{L}\{t^3\} = \frac{3!e^{-7s}}{s^4} = \frac{6e^{-7s}}{s^4}$
6. Write $v(t) = 5 + u_1(t)(t-5)$.
   $\mathcal{L}\{5 + u_1(t)(t-5)\} = \frac{5}{s} + \mathcal{L}\{u_1(t)(t-5)\} = \frac{5}{s} + e^{-s}\mathcal{L}\{t-4\} = \frac{5}{s} + e^{-s}\left(\frac{1}{s^2} - \frac{4}{s}\right) = \frac{5}{s} + \frac{e^{-s}}{s^2} - \frac{4e^{-s}}{s}$
7. Write $w(t) = 2 + u_4(t)(t^2-2)$.
   $\mathcal{L}\{2 + u_4(t)(t^2-2)\} = \frac{2}{s} + \mathcal{L}\{u_4(t)(t^2-2)\} = \frac{2}{s} + e^{-4s}\mathcal{L}\{(t+4)^2-2\}$
   . $= \frac{2}{s} + e^{-4s}\mathcal{L}\{t^2+8t+14\} = \frac{2}{s} + e^{-4s}\left(\frac{2}{s^3} + \frac{8}{s^2} + \frac{14}{s}\right)$.