Math 307 - Homework 2 - Dr. Loveless
DUE Wednesday, April 15

The problem numbers refer to the 10th edition of the book. Hand in your work in the order it is assigned (Staple all your work together before coming to class). This is a minimal list of problems, I strongly encourage you to do more problems than are assigned.

1. 2.3/4, 6(c), 9, 16, 17, 20(a), 21(a)
2. 2.4/1(see notes), 2, 3, 6, 15(see notes), 22(b)(see notes)
3. 2.5/3, 5, 7, 18, 19

NOTES AND SPECIAL INSTRUCTIONS :

- For the questions that ask 'Plot the graph', use some graphing software (wolfram alpha, graphing calculator, or the slope field plotter), then include a very rough hand-drawn sketch on your homework just to prove that you did graph it somehow.
- On 2.3/6(c): Read parts (a) and (b). You start by solving the differential equations from 6(b) with $A(h) = \pi$, $\alpha = 0.6$, $a = 0.01\pi$ and $g = 9.8$. Then answer the question.
- On 2.4/1, 2, 3, 6: You are being asked to find the interval you get when you use the theorems from this section (The solutions may actually exist in larger intervals, you are only being asked to give what you can conclude from the theorems you learned). You do NOT solve the differential equation! Just state the interval showing your work for how you rearranged the equation and found the discontinuity points.
- On 2.4/15: The questions is asking:
  - Solve the differential equation (write your answers in explicit form).
  - If $y(0) = 0$, what is the solution? What is the domain of this solution?
  - If $y(0) = y_0 > 0$, what does the solution look like (will be in terms of t and y_0)? What is the domain of the solution (in terms of y_0)?
  - If $y(0) = y_0 < 0$, what does the solution look like (will be in terms of t and y_0)? What is the domain of the solution (in terms of y_0)?
- On 2.4/22(b): You'll have to read part (a) first. But you only need to do part (b). (I just want you to read Theorem 2.4.2 and explain which hypothesis is false for the given equation and initial condition).

HINTS:

- On 2.3/4: Start by finding the formula for the volume of the tank, $V(t)$, (it starts at 200 gallons and goes up 1 gal/min).
- On 2.3/20(a), 21(a): For consistency, I suggest you make up positive and down negative (for velocity). You are modeling the behavior on the way up, so BOTH gravity and air resistance give negative forces.
- On 2.5/3, 5: Read the instructions! You are NOT solving. You are finding and classifying the equilibrium points. Also make a rough sketch (something that looks like Figure 2.5.8 from the book).
- On 2.5/18, 19: You do NOT need to solve the differential equations to answer the questions! For part (a), just translate the assumption into a differential equation (then rewrite it so it looks like what is written). The other parts can be quickly answered directly from the differential equation.

EXTRA PROBLEMS: These problems are NOT due (and they are NOT extra credit). I ask you to read these problems for your own interest and to help you to gain more exposure to differential equations applications:

- 2.3/1-5,19 are all mixing problems, try them out for more practice.
- 2.3/7-12 are all compound interest with withdrawals or deposits (loans/morgages, savings accounts).
- 2.3/13 is radio-carbon dating.
- 2.3/14, 15 are population problems.
- 2.3/16-18 are problems about temperature change. (Newton's law of cooling and other models).
- 2.3/20-31 are velocity problems (with resistance). Problem 23 is an interesting sky-diver problem (models before and after parachute opens). Problems 30 and 31 are the paths of batted baseballs.
- 2.3/27 is about velocity of an object under a liquid (there is an additional buoyant force).
- 2.3/32 is about the famous Brachistochrone problem where you are trying to find the best way to design a slide; so that gravity takes you down the slide from one point to another as fast as possible (the answer is the 'cycloid').
- 2.5/16-17 are about the Gompertz population model for restricted growth.
- 2.5/18 is about evaporation from a small pond.
- 2.3/6 and 2.5/19 are about liquids leaking out through a small hole in a container.
- 2.5/20-21 are about harvesting fish.
- 2.5/22-24 are about modeling the spread of a contagious disease.
- 2.5/28 is about chemical reactions.

You can find many more applied examples by searching online for 'differential equations applied examples'. Note that these are all first order equations, there are also many applications of second order equations which we will discuss later in the term (in particular, we will discuss various types of wave applications).