Math 307 - Homework 5 - Dr. Loveless

The problems refer to the 10th edition of the book. Hand in your work in the order it is assigned (Staple all your work together before coming to class). This is a minimal list of problems, I strongly encourage you to do more problems than are assigned.

1. 3.5/2, 3, 6, 8, 16
2. 3.6/1, 5, 7, 29 (see instructions)
3. 3.7/2, 5, 6, 11, 12, 17

NOTES AND SPECIAL INSTRUCTIONS :

- On 3.6/1: You have two tasks:
  1. Solve using the formulas below.
  2. Solve using undetermined coefficients. (You should get the same answer!)
- On 3.6/5, 7: Find your particular solution by using the variation of parameters theorem. In other words, use the 'shortcut' formulas:

$$Y(t) = u_1(t)y_1(t) + u_2(t)y_2(t), \quad \text{where} \quad u_1(t) = -\int \frac{g(t)y_2(t)}{W(y_1, y_2)}\,dt \quad \text{and} \quad u_2(t) = \int \frac{g(t)y_1(t)}{W(y_1, y_2)}\,dt,$$

- 3.6/29: Use the reduction of order method from 3.4. That is, start with the one given homogenous solution $y_1(t) = t$, write $y(t) = u(t)t$ and substitute into $t^2y'' - 2ty' + 2y = 4t^2$. You can deduce the form of $u(t)$ and from that you will get a second homogeneous solution AND a particular solution. In the end, give the general solution for the differential equation.

  Side comment about problem 3.6/29: One might ask, why we don't always use the guess $y(t) = u_1(t)y_1(t)$ instead of $y(t) = u_1(t)y_1(t) + u_2(t)y_2(t)$. In other words, why not just take $u_2(t) = 0$ since it is simper and we saw that it works in 3.6/29. The advantage of the general choice $y(t) = u_1(t)y_1(t) + u_2(t)y_2(t)$ is that you get a nice general form in terms of relatively simple to state (somewhat symmetric looking) integrals for $u_1(t)$ and $u_2(t)$. In general, the method of 3.6/29 tends to lead to messier integrals, but in this particular case you were (hopefully) able to integrate.