Print Your Name

Problem	Total Points	Score
1	10	
2	10	
3	15	
4	15	
5	10	
Total	60	

You should:

- write complete solutions or you may not receive credit.
- box your final answer.
- check that your exam contains a total of 5 pages.

You may:

- use ten sheets of notes and a calculator.
- write on the backs of the pages if you need more room.

Please do not:

- come to the front of the room to ask questions (raise your hand).
- share notes or calculators.
- use any electronic device other than a calculator.

Signature. Please sign below to indicate that you have not and will not give or receive any unauthorized assistance on this exam.

Signature: _____

1. Solve the initial value problem

$$-ty' = (y-5)^2, \quad y(1) = 6.$$

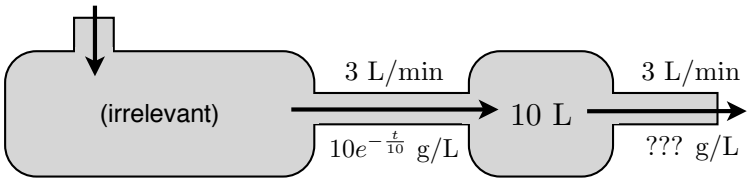
Your solution only needs to be valid close to $t = 1$, so you can assume $t > 0$.

2. Find the general solution to the differential equation

$$ty' = at - 2y, \quad t > 0, \quad a \text{ some constant.}$$

3. Two connected tanks are filled with a mixture of green dye and water. Water flows into the first, is mixed, the mixture flows into the second, is mixed again, and then drains. Suppose that water is flowing through these tanks at a rate of 3 L/min, so that the volume of water in each tank stays constant. You don't need to worry about the first tank,[1] because I already figured out that the concentration of the water flowing into the second tank is $10e^{-t/10}$ g/L. If the second tank has a volume of 10 L and an initial dye concentration of 1 g/L, find a formula for the concentration of dye in the second tank.

   Please box your differential equation and your final answer.



[1] In case you are interested, the first tank has a volume of 30 L, and fresh water is flowing into it. It has an initial concentration of 10 g/L of dye.

4. A population of bacteria reproduce at an hourly rate of r (proportional to the current population) and die off at a constant rate d per hour (regardless of what the current population is). The initial population of bacteria is P_0. Write and solve an initial value problem modeling this situation. Box both the differential equation and the final answer.

5. (a) Show that $y = \sin(t)$ is a solution to the differential equation $y'' + y = 0$.

(b) Give an example of a first-order differential equation that is neither linear nor separable. Do not solve it.

(c) Find the equilibrium points of the differential equation $y' = (y-1)(y-2)(y-3)$ and classify them as stable, unstable, or neither.