1. This differential equation is separable. We integrate to get

$$-\int \frac{dy}{(y-5)^2} = \int \frac{dt}{t}$$
$$\frac{1}{y-5} = \ln t + c.$$

Using our initial value, we find that $c = 1$. Solving for y, we get

$$y = \frac{1}{\ln t + 1} + 5.$$

2. This is linear, so we put it into the proper form

$$y' + \frac{2}{t}y = a.$$

We find the multiplying factor

$$\mu(t) = e^{2\ln t} = t^2.$$

So we have

$$t^2 y = \int (at^2\, dt) = \frac{at^3}{3} + c.$$

Solving for y, we get

$$y = \frac{at}{3} + \frac{c}{t^2}.$$

3. If y denotes the mass of dye in the tank, then salt flows in at a rate of $30e^{-t/10}$ and out at a rate of $3y/10$. So the initial value problem we want to solve is

$$y' = 30e^{-t/10} - 3y/10, \quad y(0) = 10.$$

This is a linear differential equation, and the multiplying factor is $e^{3t/10}$. So we get

$$y = e^{-3t/10}\int 30e^{t/5}\, dt = e^{-3t/10}(150e^{t/5} + c) = 150e^{-t/10} + ce^{-3t/10}$$

The constant is -140, but then we need to divide everything by volume (= 10 L) to get concentration of dye. The final answer is

$$y = 15e^{-t/10} - 14e^{-3t/10}.$$

4. The differential equation is $y' = ry - d$. It is linear and separable, so you can use either method to find the solution $y = d/r + ce^{rt}$. The initial condition gives $c = P_0 - d/r$, so the final solution is

$$y = \frac{d}{r} + \left(P_0 - \frac{d}{r}\right)e^{rt}.$$

5. (c) The equilibrium points are y=1, 2, and 3. The equilibrium $y = 2$ is stable, and the other two are unstable.