Math 307K, Winter 2012 **Midterm** **University of Washington**

Your Name

Student ID #

- This exam is closed books. No aids are allowed for this exam. You can use any information on the note sheet on the last page without justification.
- In order to receive credit, you must **show all of your work**; to obtain full credit, you must provide mathematical justifications. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct.
- Give your answers in exact form (for example $\frac{\pi}{3}$ or $5\sqrt{3}$).
- If you need more room, use the backs of the pages and indicate that you have done so.
- Raise your hand if you have a question.
- This exam has 5 pages, plus this cover sheet and the note sheet. Please make sure that your exam is complete.
- You have 50 minutes to complete the exam.

Question	Points	Score
1	5	
2	4	
3	6	
4	6	
5	3	
Total	24	

1. (5 total points)

   (a) (4 points) Suppose the position of some object at time t is described by the following initial value problem:

   $$y' = \frac{1}{ty}, \quad t > 0, \quad y(1) = 1.$$

   Determine the position of the object at time t.

   (b) (1 point) For which times t is the solution defined?

2. (4 points) Determine explicitly all the solutions to the differential equation

$$(1+t^2)y' + y = 1.$$

3. (6 points) Initially, a tank contains 6 gal of water containing 1 lb of salt. There is water flowing into the tank through two pipes: Water containing salt is entering the tank through the first pipe at rate of 2 gal/min. Several measurements indicate that the amount of salt contained in one gallon of the incoming water is $e^{-\frac{3}{2}t}$ lb at time t. One gallon of fresh water per minute is entering the tank through the second pipe. Finally, the well-stirred mixture is draining the tank at a rate of 3 gal/min.

   Determine the amount of salt at any time $t \geq 0$.

4. (6 total points) Suppose that fish are harvested a constant rate E from the total population. Then we have to modify the logistic equation as follows:

$$\frac{\mathrm{d}P}{\mathrm{d}t} = r\left(1 - \frac{P}{K}\right)P - EP$$

where P is a function of time and represents the number of the fish at time t, K and r are positive constants, and $E \geq 0$ is a nonnegative constant.

(a) (5 points) Assume that $E < r$. Determine and classify all the equilibrium solutions to this equation.

(b) (1 point) How does the number and classification of the equilibrium solutions change if we assume $E > r$?

5. (3 points) A kid places a skyrocket in a bottle, burns the fuse, and runs away. After 5 seconds, the skyrocket launches straight into the sky. The rocket weights 0.4 kg, including 0.2 kg of propellant. For that rocket, the force due to air resistance has been measured to be $|v|/60$ N. After ignition, the propellant burns down at a constant rate of 0.04 kg/s and thereby exerts a constant force of 20 N. Once all the propellant is burnt, the skyrocket explodes.

   Write down, but do NOT solve, an initial value problem for the velocity of the skyrocket up to the time where it explodes, as well as the time interval for which the differential equation is valid.