Math 307A, Midterm 1 Name: ______________________

Spring 2013

Instructions.

- **DON'T PANIC!** If you get stuck, take a deep breath and go on to the next question. Come back to the question you left if you have time at the end.
- There are 4 questions on 6 pages. Make sure your exam is complete.
- You are allowed one double-sided sheet of notes in your own handwriting. You may not use someone else's note sheet.
- You may use a simple scientific calculator, but you don't need to. No fancy calculators or other electronic devices allowed. If you didn't bring a simple calculator, then just don't use a calculator.
- It's fine to leave your answers in exact form. If you use a calculator, approximate to two decimal places.
- **Show your work**, unless instructed otherwise. If you need more space, raise your hand and I'll give you some extra paper to staple onto the back of your test.
- Don't cheat. If I see that you aren't following the rules, I will report you to UW.

Question	Points	Score
1	17	
2	10	
3	10	
4	11	
Total:	48	

1. (a) (5 points) Solve the IVP (find an explicit formula for y), and find the interval on which your solution is valid.

$$\frac{dy}{dx} = (1 - 2x)y^2, \qquad y(0) = -\frac{1}{2}$$

(b) (5 points) Solve the differential equation. You may leave your answer in implicit form: no need to solve for y.

$$3x^2 + y + (x + 2y)\frac{dy}{dx} = 0.$$

(c) (7 points) Consider the nonseparable, nonlinear differential equation

$$\frac{dy}{dt} + \frac{2}{t}y = \frac{y^3}{t^2}, \qquad t > 0.$$

It is called a *Bernoulli equation.*

Use the substitution $v = y^{-2}$ to solve the equation. Leave it in general form, with the constant C. Be sure your final answer has the variables y and t only, no v. You may leave your answer in implicit form: no need to solve for y.

2. Consider the differential equation

$$y' = 2y + 3e^t.$$

(a) (4 points) Find the general form of the solution $y(t)$.

(b) (6 points) Find the solution $y(t)$ that is tangent to the horizontal line $y = -1$.

3. Read the instructions here carefully, so you avoid doing extra work!

   (a) (5 points) A tank has 80 liters (L) of water with 15 grams (g) of salt dissolved in it. At time $t = 0$ water with 20 g/L of salt flows into the tank at a rate of $\frac{1}{4}$ L/s. At the same time, a drain opens in the bottom of the tank and the well-mixed solution drains out at 1 L/s.

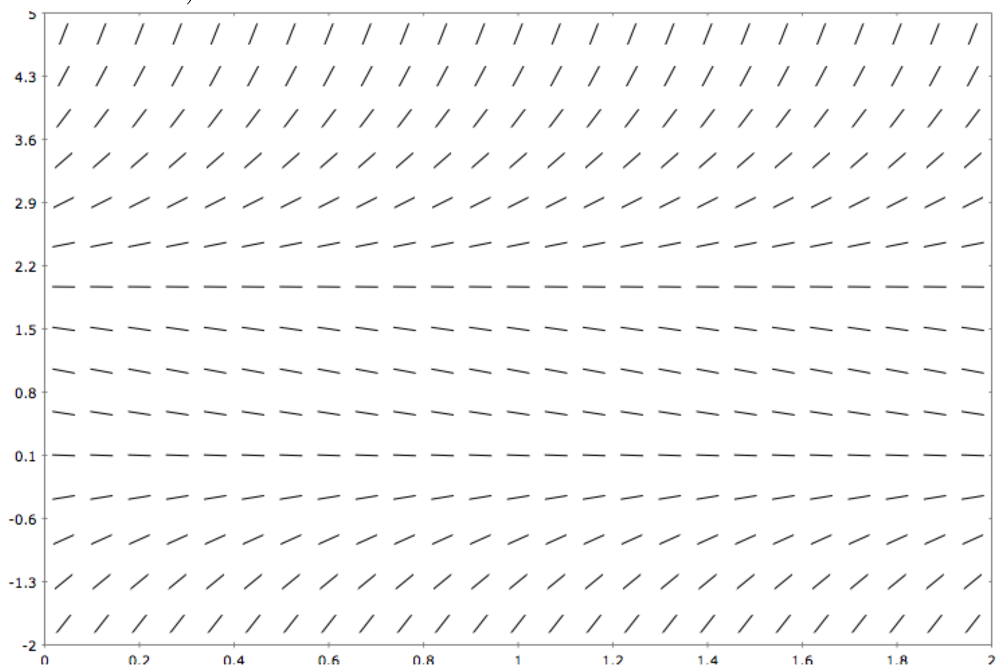
   Let $Q = Q(t)$ be the quantity of salt in the tank in grams. Write a differential equation relating Q, t (in seconds), and Q'. Be sure there are no other variables in your expression. **You don't have to solve the equation**; just set it up.

   (b) (5 points) Let $P = P(t)$ be the number of bacteria in a certain area, where t is in days. The population is modeled by the differential equation

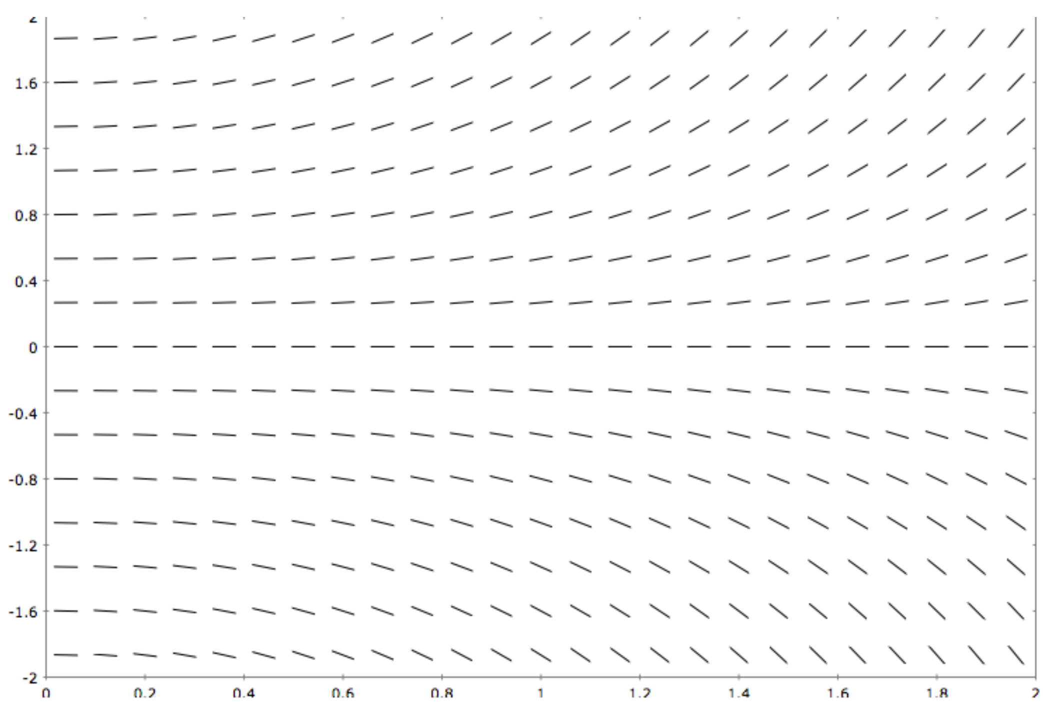
$$P' = rP$$

   for some r. The population is dying off: every five days, the population is cut in half. What is r?

4. (a) (4 points) Each of the two slope fields below has a list of differential equations below it. Circle the DE that matches the slope field. (t is the horizontal axis; y is the vertical axis.)



$y' = y(y-2)$ $\qquad$ $y' = -y(y-2)$ $\qquad$ $y' = y(y+2)$ $\qquad$ $y' = -y(y+2)$



$y' = y$ $\qquad$ $y' = -y$ $\qquad$ $y' = ty$ $\qquad$ $y' = -ty$

(b) (7 points) Consider the differential equation

$$\frac{dy}{dt} = (y^2 - 1)(y + 2)$$

Draw a coordinate plane below. Label the axes, and sketch at least ten solutions to the differential equation.

Read this carefully!: Include all equilibrium solutions. Make sure your solutions start at $t = 0$ (or before), and draw them for long enough so that their eventual behavior is clear to me. Include as many different behaviors as possible.