Your Name

Student ID #

1	10	
2	10	
3	10	
4	10	
5	10	
Total	50	

- Complete all questions. BOX your answers. Do not write outside the marginal lines.
- One handwritten two-sided sheet of note and calculator are allowed. NO CHEATING!
- In order to receive credit, you must **show all of your work**; to obtain full credit, you must **provide mathematical justifications**. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct.
- Raise your hand if you have a question.
- You have 50 minutes to complete the midterm.

$$\int x^a \,\mathrm{d}x = \tfrac{x^{a+1}}{a+1} \qquad \int \tfrac{1}{x}\,\mathrm{d}x = \ln|x|$$
$$\int e^x \,\mathrm{d}x = e^x \qquad \int a^x \,\mathrm{d}x = \tfrac{a^x}{\ln a}$$
$$\int \sin x \,\mathrm{d}x = -\cos x \qquad \int \cos x \,\mathrm{d}x = \sin x$$
$$\int \sec^2 x \,\mathrm{d}x = \tan x \qquad \int \sec x \tan x \,\mathrm{d}x = \sec x$$
$$\int \csc x \cot x \,\mathrm{d}x = -\csc x \qquad \int \csc^2 x \,\mathrm{d}x = -\cot x$$
$$\int \sec x \,\mathrm{d}x = \ln|\sec x + \tan x| \qquad \int \csc x \,\mathrm{d}x = \ln|\csc x + \cot x|$$
$$\int \tan x \,\mathrm{d}x = \ln(\sec x) \qquad \int \cot x \,\mathrm{d}x = \ln(\sin x)$$
$$\int \sinh x \,\mathrm{d}x = \cosh x \qquad \int \cosh x \,\mathrm{d}x = \sinh x$$
$$\int \tfrac{\mathrm{d}x}{x^2+a^2} = \tfrac{1}{a}\arctan\left(\tfrac{x}{a}\right) \qquad \int \tfrac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\left(\tfrac{x}{a}\right)$$
$$\int \tfrac{\mathrm{d}x}{x^2-a^2} = \tfrac{1}{2a}\ln\left|\tfrac{x-a}{x+a}\right| \qquad \int \tfrac{\mathrm{d}x}{\sqrt{x^2\pm a^2}} = \ln\left|x \pm \sqrt{x^2+a^2}\right|$$

1.a (4points) Find a differential equation whose general solution is $y = c_1 e^{-2t}\cos(2t) + c_2 e^{-2t}\sin(2t)$.

1.b.(3 pts) Find a differential equation whose general solution is $y = c_1 e^{-2t}\cos(2t) + c_2 e^{-2t}\sin(2t) + \sin(2t)$.

1.c. (3pts) Find a differential equation whose general solution is $y = c_1 e^{-2t} + c_2 t e^{-2t}$.

2. (10 points) Solve the following initial value problem:

$$y'' + 2y' + 2y = (5t - 1)e^t + 3, \quad y(0) = y'(0) = 1.$$

This page is intentionally left blank. You can write your solution here.

3. (10 pts) A mas that weighs 8 lb stretches a spring 6 in. The system is acted on by an external force of $8\sin(8t)$ lb. If the mass is pulled down 3 in and then released, determine the position of the mass at any time. There is no damped force.

4. (10 pts) Given $y_1(t) = t$ satisfying the following differential equation, find a second solution of this equation:

$$t^2y'' - t(t+2)y' + (t+2)y = 0, \;\; t > 0.$$

5. (10 pts) Find the general solution of the following differential equation:

$$t^2y'' - t(t+2)y' + (t+2)y = t^3 \sin t, \quad t > 0.$$