Your Name

Student ID #

- Cellphones off please!
- You are allowed one two-sided handwritten notesheet for this midterm. You may use a scientific calculator; graphing calculators and all other course-related materials may not be used.
- In order to receive credit, you must **show all of your work**. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct.
- Give your answers in exact form (for example $\frac{\pi}{3}$ or $e^{-5\sqrt{3}}$) unless explicity stated otherwise by the question.
- If you need more room, use the backs of the pages and indicate that you have done so.
- Raise your hand if you have a question.
- This exam has 6 pages, plus this cover sheet. Please make sure that your exam is complete.
- You have 50 minutes to complete the exam.

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total	50	

1. (10 points) Solve the following initial value problem explicitly. Your answer should be a function in the form $y = g(t)$, where there is no undetermined constant in g.

$$\frac{dy}{dt} = \frac{t}{y + t^2 y}, \quad y(0) = -3.$$

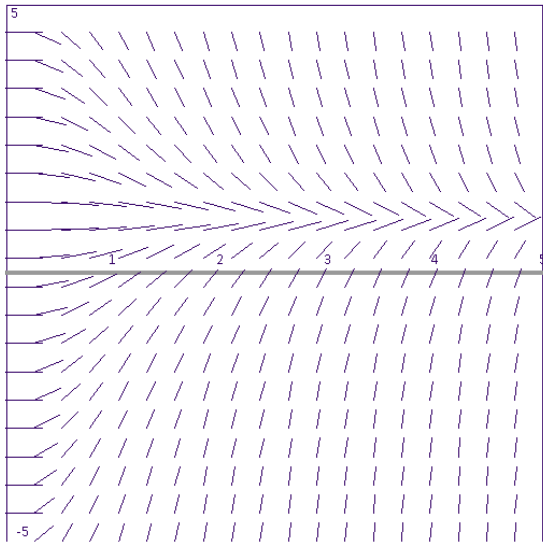
2. (10 total points) Consider the following linear initial value problem:

$$(t^2 - 4t - 5)\frac{dy}{dt} + 2(t+1)y - 1 = 0, \quad y(0) = y_0.$$

(a) (3 points) Using the existence and uniqueness theorem for linear differential equations, state the maximum interval on which a unique solution to the above IVP is guaranteed to exist.

(b) (7 points) Solve the above differential equation for the case $y_0 = -\frac{1}{5}$.

3. (10 total points)
The slope field to the differential equation $\frac{dy}{dx} = f(x,y)$ is plotted below for $0 \leq x \leq 5, -5 \leq y \leq 5$:



(a) (4 points) Circle the differential equation that corresponds to the above slope field.

$$\frac{dy}{dx} = x(y+1) \qquad \frac{dy}{dx} = -x(y+1) \qquad \frac{dy}{dx} = x(y-1) \qquad \frac{dy}{dx} = -x(y-1)$$

(b) (6 points) Let $y = \phi(x)$ be the solution to the differential equation you circled above that satisfies the initial condition $y(0) = 0$. Use Euler's method with a step size of $h = 0.5$ to estimate the value of the solution at $x = 1.5$. You may use decimal approximations in your final answer, but if you do keep at least 3 digits precision at all points.

4. (10 total points) Consider the autonomous differential equation

$$\frac{dy}{dt} = sin^2(y) - K,$$

where K is a constant such that $y = \frac{\pi}{3}$ is an equilibrium solution.

(a) (5 points) Find K, and state whether $y = \frac{\pi}{3}$ is a stable, unstable or semistable equilibrium solution. Be sure to justify your answer.

(b) (5 points) Suppose $y = \phi(t)$ is the unique solution to the differential equation satisfying the initial condition $y(0) = 0$. Find $\lim_{t\to\infty} \phi(t)$.

5. (10 total points + 3 bonus points) Water hyacinth is a particularly aggressive invasive plant species in lakes in the southern US. One of the reasons is that it grows very quickly: under good conditions a population will grow at a rate proportional to its own size, with its biomass increasing by a factor of $e = 2.71828\ldots$ every 14 days. Suppose a water hyacinth population establishes itself in a large lake in Florida where conditions are close to ideal. When ecologists discover the population it has a biomass of 750kg.

   Removal efforts begin immediately; however, because it takes some time to train local volunteers to remove the weed efficiently, the rate $R(t)$ at which water hyacinth can be removed from the lake is given by the function
   $$R(t) = 600\left(1 - e^{-t}\right),$$
   where t is measured in **weeks** since the beginning of the removal effort, and $R(t)$ is in kg/week.

   (a) (7 points) Establish an initial value problem and solve it to find an explicit formula for the biomass of water hyacinth in the lake at time t.

[Question continued overleaf]

(b) (3 points) Will efforts to completely remove the water hyacinth from the lake be successful? Justify your answer.

(c) (Bonus: 3 points) If the answer to the above question is yes, estimate how many weeks it will take for the water hyacinth to be removed completely from the lake. If the answer to the above question is no, estimate how many weeks it will take for the water hyacinth to reach 10000kg biomass. You may use decimal approximations in your final answer (but keep at least 4 digits precision at all points).