Math 307 - Spring 2015
Exam 2
May 20, 2015

Name: ______________________________

Section: ______________________________

Student ID Number: ______________________________

- There are 5 pages of questions. Make sure your exam contains all these questions.
- You are allowed to use a scientific calculator (**no graphing calculators and no calculators that have calculus capabilities**) and one **hand-written** 8.5 by 11 inch page of notes.
- You must show your work on all problems. The correct answer with no supporting work may result in no credit. **Put a box around your FINAL ANSWER for each problem and cross out any work that you don't want to be graded.** Give exact answers wherever possible.
- If you need more room, use the backs of the pages and indicate to the grader that you have done so.
- Raise your hand if you have a question.
- There may be multiple versions of the exam so if you copy off a neighbor and put down the answers from another version we will know you cheated. Any student found engaging in academic misconduct will receive a score of 0 on this exam. All suspicious behavior will be reported to the student misconduct board. In such an instance, you will meet in front of a board of professors to explain your actions.
  DO NOT CHEAT OR DO ANYTHING THAT LOOKS SUSPICIOUS!
  WE WILL REPORT YOU AND YOU MAY BE EXPELLED!
- You have 50 minutes to complete the exam. Budget your time wisely.
  SPEND NO MORE THAN 10 MINUTES PER PAGE!

PAGE 1	8	
PAGE 2	10	
PAGE 3	10	
PAGE 4	12	
PAGE 5	10	
Total	50	

GOOD LUCK!

1. (8 points) The differential equation of a given mass-spring system is $4u'' + \gamma u' + 9u = 0$ with initial conditions $u(0) = 2$ and $u'(0) = 0$. You are told that the system is **critically damped**.
Solve the initial value problem to find $u(t)$. (Hint: First find γ).

2. (10 pts) **Parts (a) and (b) below are independent!**

   (a) A 5 pound object stretches a spring 3 inches beyond its natural length (and is at rest). The damping force is 2 lbs when the upward velocity is 10 ft/s. There is no external forcing. Initially, the mass is pulled downward 6 in and given an initial upward velocity of 1 ft/s. Set up the differential equation AND initial conditions for the displacement $u(t)$.
   (DO NOT SOLVE)

   (b) For a different mass-spring system the differential equation is $2u'' + 14u = F(t)$, where $F(t)$ is the forcing function and t is in seconds.

      i. Assume there is no forcing ($F(t) = 0$). The mass is pulled down to some starting point. The mass is released. How long will it take to first get back to its starting point?

      ii. Let ω_0 be the natural frequency of the unforced system.
      Assume $F(t) = 5\cos(\omega t)$ where ω is the frequency of the forcing.
      There is a significant difference in long-term behavior of the solutions depending on whether $\omega = \omega_0$ or $\omega \neq \omega_0$. Briefly in words, give the name of this behavior AND specifically describe the key difference in behavior.
      (You don't have to solve anything here, one to two sentences is all you need).

3. (10 pts) The charge $Q(t)$ at time t on the capacitor in a particular RLC circuit satisfies

$$Q'' + Q' + \frac{5}{4}Q = \frac{37}{4}\cos(2t).$$

A particular solution is given by $Y(t) = -\frac{11}{5}\cos(2t) + \frac{8}{5}\sin(2t)$. (You do not have to find this!)

(a) Give the general solution for $Q(t)$.

(b) Draw a rough sketch of the steady state response.
Find and clearly label the amplitude and period. Also label the y-intercept.

4. (12 pts) Note: The equation $r^2 - r - 2 = 0$ has the roots $r_1 = 2$ and $r_2 = -1$.

(a) Use undetermined coefficients to find a particular solution to $y'' - y' - 2y = 5 + 3e^{2t}$.

(b) Use undetermined coefficients to find a particular solution to $y'' - y' - 2y = \cos(t)$.

5. (10 pts) Consider the differential equation

$$y'' - \frac{6}{t}y' + \frac{6}{t^2}y = 5t^3 \quad \text{with} \quad t > 0.$$

The function $y_1(t) = t$ is a solution to the corresponding homogeneous equation. Use reduction of order to find the general solution.