Math 307 A - Spring 2015
Final Exam
June 10, 2015

Name: ______________________________

Student ID Number: ______________________________

- There are 8 pages of questions. In addition, the last page is the basic Laplace transform table. Make sure your exam contains all these pages.
- You are allowed to use a scientific calculator (**no graphing calculators and no calculators that have calculus capabilities**) and one **hand-written** 8.5 by 11 inch page of notes.
- You must show your work on all problems. The correct answer with no supporting work may result in no credit. **Put a box around your FINAL ANSWER for each problem and cross out any work that you don't want to be graded.** Give exact answers wherever possible.
- If you need more room, use the backs of the pages and indicate to the grader that you have done so.
- Raise your hand if you have a question.
- There may be multiple versions of the exam so if you copy off a neighbor and put down the answers from another version we will know you cheated. Any student found engaging in academic misconduct will receive a score of 0 on this exam. All suspicious behavior will be reported to the student misconduct board. In such an instance, you will meet in front of a board of professors to explain your actions.
DO NOT CHEAT OR DO ANYTHING THAT LOOKS SUSPICIOUS!
WE WILL REPORT YOU AND YOU MAY BE EXPELLED!
- You have 110 minutes to complete the exam.

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GOOD LUCK!

1. (13 pts)

(a) Find the general explicit solution to $ty' - 2y = t^6$.

(b) Find the explicit solution to $y' = 4xy^2e^{2x}$ with $y(0) = 4$.

2. (14 pts)

(a) Consider $y' = (1 - 2y)\,(y - 4)^2$.

i. Determine the critical (equilibrium) points and classify each one as stable, unstable or semistable.

ii. Let $y(t)$ be the solution that satisfies the given differential equation with the initial condition $y(1) = 3$.
Use Euler's method with $h = 0.1$ to approximate the value of $y(1.1)$.

(b) A baseball is dropped from an airplane. The mass of a baseball is about 0.2 kg. The force due to air resistance is proportional, and in opposite direction, to velocity with proportionality constant k (where $k > 0$).

Just like we did in homework, assume there are two forces acting on the ball: the force due to gravity and the force due to air resistance. (Recall: Newton's second law says $ma = F$ and the acceleration due to gravity is 9.8 meters/second^2.)

i. Give the differential equation and initial conditions for the velocity $v(t)$. (Do not solve)

ii. The value of $\lim\limits_{t\to\infty} v(t)$ is called the terminal velocity. For a baseball, terminal velocity is known to be about 42 meters/second. Using this fact, find the value of the proportionality constant k. (Hint: You do NOT need to solve the differential equation).

3. (10 pts) Some cookie dough with an initial temperature of 40 degrees Fahrenheit is placed in an oven and the oven is turned on. The temperature of the oven is given by $f(t) = 350 - 280e^{-t/2}$ degrees Fahrenheit where t is in minutes. Assume the differential equation for the temperature of the cookie dough, $y(t)$, is given by

$$\frac{dy}{dt} = -\frac{1}{2}\left(y - 350 + 280e^{-t/2}\right).$$

Solve the differential equation to find the temperature of the cookie dough, $y(t)$, at time t minutes. (Hint: It's linear!)

4. (12 pts) A certain mass-spring system satisfies $mu'' + 3u' + u = 0$, where m is the mass of the object attached to the end of the spring. The initial conditions are $u(0) = 3$ and $u'(0) = 0$.

   (a) For what masses, m, will the system exhibit (damped) oscillations?
   (Your answer will be a range of values)

   (b) Find the quasi-period of the solution if $m = 5$.

   (c) Find the solution $u(t)$ if $m = 2$. (Use the initial conditions.)

5. (13 pts) Give the solution to $y'' + 4y = te^{2t}$ with $y(0) = 0$ and $y'(0) = 0$.

6. (14 pts)

(a) Give the form of a particular solution to $y'' - 4y' + 4y = 5 + te^{2t}$. (Do not solve, just give the form you would use for undetermined coefficients. Your answer will involved 'A, B, ...').

(b) Use the Laplace transform table (and step functions) to answer these questions:

i. Find the Laplace transform, $\mathcal{L}\{f(t)\}$, for the function $f(t) = \begin{cases} 3 & , 0 \leq t < 6; \\ t + \cos(t-6) & , t \geq 6. \end{cases}$

ii. Find the inverse Laplace transform $\mathcal{L}^{-1}\left\{e^{-2s}\frac{2}{s^3} - e^{-5s}\frac{4}{s-8}\right\}$.

7. (12 pts) Use the Laplace transform table (and algebra) to answer these questions:

(a) Find the Laplace transform of both sides of $y' = 3te^{4t} + 2t^3$ with $y(0) = 5$ and solve for $\mathcal{L}\{y\}$. (Don't do partial fraction and don't solve for $y(t)$, just stop when you get $\mathcal{L}\{y\}$ by itself and the other side all in terms of s.)

(b) Find the inverse Laplace transform, $\mathcal{L}^{-1}\left\{\frac{1}{s^2(s-2)} + \frac{1}{s^2+6s+13}\right\}$.

8. (12 pts) Solve $y'' + y = \begin{cases} 1 & , 0 \le t < 3; \\ 5 & , t \ge 3. \end{cases}$ with initial conditions $y(0) = 0$, $y'(0) = 2$.

Laplace Transform Table for Final Exam - Dr. Loveless

$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$
1	$\frac{1}{s}$
e^{at}	$\frac{1}{s-a}$
$\cos(bt)$	$\frac{s}{s^2+b^2}$
$\sin(bt)$	$\frac{b}{s^2+b^2}$
$e^{at}\cos(bt)$	$\frac{s-a}{(s-a)^2+b^2}$
$e^{at}\sin(bt)$	$\frac{b}{(s-a)^2+b^2}$
t^n	$\frac{n!}{s^{n+1}}$
$t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}}$
$u_c(t)$	$\frac{e^{-cs}}{s}$
$u_c(t)f(t-c)$	$e^{-cs}F(s)$