Math 307 - Homework 3 - Dr. Loveless
DUE Wednesday, April 27

The problem numbers refer to the 10th edition of the book. Hand in your work in the order it is assigned (Staple all your work together before coming to class). This is a minimal list of problems, I strongly encourage you to do more problems than are assigned.

1. 3.1/1, 2, 5, 8, 9, 12, 16, 17, 20, 21
2. 3.2/1, 4, 28, 41, 45
3. Also complete the following problems. (Read the hints first).
   Combine, expand or simplify each of the following into the form $a + bi$. (*i.e.* Find a and b).
   (a) $(3 - 4i) + (10 + 9i)$
   (b) $(5 + 7i)(2 - 4i)$
   (c) $i^2 + i^3 + i^4 + i^5$
   (d) $\frac{1}{3+2i}$ (Hint: Multiplying the top and bottom by the conjugate, which is $3 - 2i$).
   (e) $e^{\frac{\pi}{3}i}$
   (f) $e^{\pi i}$
   (g) $\text{Exp}(2 - \frac{\pi}{2}i)$

HINTS :

- The problems in 3.1 should be quick to solve. See lecture, textbook and review examples. If you are having trouble, come ask in office hours.
- 3.2/28: In part (a), check that the two functions are solutions (show me your derivatives and your checking). Also verify they form a fundamental set of solutions.
  In part (b), use a theorem to quickly answer this question. Note: all together you have the five functions: $y_1(t) = e^{-t}$, $y_2(t) = e^{2t}$, $y_3(t) = -2e^{2t}$, $y_4(t) = e^{-t} + 2e^{2t}$, $y_5(t) = 2e^{-t} + 4e^{2t}$.
  In part (c), you are checking if the given pairs form fundamental sets of solutions.
  Recall: Two functions $f(t)$ and $g(t)$ form a *fundamental set of solutions* when the Wronskian of f and g is not zero.
- 3.2/41: Use the product rule to expand $[P(x)y']' + [f(x)y]' = 0$. In order for this to be the same as $P(x)y'' + Q(x)y' + R(x)y = 0$, what must $f(x)$ and $f'(x)$ be equal to? Then you should get the result.
- 3.2/45: This is exact; indeed $P(x) = x^2$, $Q(x) = x$, $R(x) = -1$ and $P''(x) - Q'(x) + R(x) = 0$. From the last part you will know that $f(x) = Q(x) - P'(x)$, get this function. Now set up $[P(x)y']' + [f(x)y]' = 0$, which will look like $\frac{d}{dx}\left(x^2 \frac{dy}{dx}\right) + \frac{d}{dx}\left(f(x)y\right) = 0$. Once you get this set up you can integrate both sides with respect to x to get $x^2 \frac{dy}{dx} + f(x)y = c$. This is a first order equation! Solve it. (I put this in here for a chance for you to review and so you could see one other method for solving second order equations).
- General comment: Sometimes the book (or I) might write $\text{Exp}(t)$ instead of e^t. This is just to make it easier to read the exponent. For example $\text{Exp}(2 - \frac{\pi}{2}i)$ is the same as $e^{2-\frac{\pi}{2}i}$.