Math 307 - Homework 5 - Dr. Loveless

Hand in your work in the order it is assigned (Staple all your work together before coming to class). **This is a minimal list of problems**, I strongly encourage you to do more problems than are assigned.

1. 3.5/2, 3, 6, 8, 16
2. 3.6/29, 30 (see instructions)
3. 3.7/2, 5, 6, 8, 11, 12, 17
4. 3.8/6

NOTES AND SPECIAL INSTRUCTIONS :

- 3.6/29, 30: Use the **reduction of order** method from 3.4. You don't need to read problem 28 and you don't need to read 3.6, you just have to do reduction of order exactly like we did in section 3.4. If you have forgotten, check out the review sheet form 3.4 which contains several examples of reduction of order. Again, start with the one given homogenous solution $y_1(t)$, write $y(t) = u(t)y_1(t)$ and find y' and y''. Substitute these into the given differential equation and simplify. This will give you a first order equation to solve (which you can solve with integrating factors).
- *Side Comment*: You are NOT expected to know 3.6 (variation of parameters) for the test. If you are curious you can read 3.6 for a general method that takes the two homogenous solution and finds a particular solution (in much the same way we did reduction of order). The method is quite general and we can derive integrals for the form of the particular solution: $Y(t) = u_1(t)y_1(t) + u_2(t)y_2(t)$, where $u_1(t) = -\int \frac{g(t)y_2(t)}{W(y_1,y_2)}\,dt$ and $u_2(t) = \int \frac{g(t)y_1(t)}{W(y_1,y_2)}\,dt$, where $W(y_1, y_2)$ is the Wronskian. The department makes this section optional for instructors, I covered it last year, but it took too much time for the derivation and added unnecessary confusion to chapter 3. By skipping section 3.6, we can do more practice with undetermined coefficients and mass-spring applications.

More homework hints on back:

Homework hints:

- In section 3.7, you will want my review on waves handy (this is what you are doing in problem 2 and elsewhere): http://www.math.washington.edu/∼aloveles/Math307Spring2016/m307Waves.pdf

- Recall:
  restorative spring force = kx (Note: this is Hooke's law and k is the spring constant).
  damping force = $-\gamma u'(t) = -\gamma$(velocity of spring) (Note: γ is the damping constant)

- Here are some reminders about units:
  In metric units, we typically use:
  Mass = kilograms = kg, Force = Newtons = N, Distance = meters = m.
  Remember that Force = (Mass)(Acceleration), so on Earth you convert from from kilograms to Newtons by multiplying by 9.8 m/s^2 (which is the acceleration due to gravity).
  Note also that 1000 g = 1 kg, 100 cm = 1 m.
  In SI units, we typically use:
  Force = pounds = lbs, Distance = feet = ft.
  In SI units, we don't have a standard, commonly used mass unit, but to get mass you divide by 32 feet/sec^2 (which is the acceleration due to gravity).
  Note also that 12 in = 1 ft.

- 3.7/5: Note: 6 in = 1/2 ft and 3 in = 1/4 ft.
  You are told that 'spring force' = 2 lbs when $x = 1/2$ foot. Since 'spring force' = kx, you can plug in this information to get k.
  You also need to know the mass which is $m = \frac{\text{force}}{\text{gravity}} = \frac{2}{32} = \frac{1}{16}$ lbs-s^2/ft. You are told there is no damping (so $\gamma = 0$). Now you should be ready to solve a second order equation.

- 3.7/6: Note: 100 g = 0.1 kg and 5 cm = 0.05 m. In these units, to get force we multiply mass by 9.8, so 0.1 kg gives 0.98 N. Now you can get k. You are told there is no damping so $\gamma = 0$.

- 3.7/8: There is no resistor, so $R = 0$ ohms and you are given $L = 1$ henry and $C = 0.00000025 = 0.25 \times 10^{-6}$ Farads. On pages 202 and 203, they show you the differential equation (with a picture). $Q(t)$ is total charge in coulombs on the capacitor at time t and $Q'(t)$ is the current in amperes. There is nothing to set up here, you are given everything, solve the second order homogeneous equation.

- 3.7/11: Follow what you did in 3.7/6, but also note that you are told that the magnitude of the damping force = 3 N when velocity is 5 m/s so you can find γ.

- 3.8/6: Just like problem 3.7/6 and 3.7/11! Only difference is there is now an external forcing term, so you'll get something of the form $mu'' + \gamma u' + ku = F(t)$ where $F(t) = 10\sin(t/2)$.