1. (5 points) Solve the initial value problem

$$y' + 2y = te^{-2t}, \qquad y(1) = 0.$$

2. (5 points) Let $y(t)$ be a solution to the differential equation

$$y' = (y-1)(y-2)^2, \qquad y(1) = y_0.$$

Determine all possible values of y_0 so that $\lim_{t\to\infty} y(t) = 2$.

3. (5 points) A tank contains 2 kg of salt disolved in 500 L of water. Fresh water flows in at a rate of r L/s, and mixed solution flows out at the same rate.

   Determine the value of r so that the amount of salt in the tank is reduced by half in exactly 1 hour.

4. (5 points) Suppose you have an object of mass 1 kg hanging from a spring with spring coefficient 8 N/m and damping coefficient 2 N/(m/s). An external force (measured in Newtons) of $F_0 \cos 2t$ is applied to the system.

   Determine the amplitude of the steady state response.

5. (5 points) Solve the initial value problem

$$y'' + 7y' + 6y = \begin{cases} 0, & 0 \le t < 2 \\ e^{-2t}, & 2 \le t \end{cases}, \quad y(0) = 3, \quad y'(0) = 7.$$

6. (5 points) Consider the initial value problem

$$y'' + 9y = A\delta_c(t), \quad y(0) = 0, \quad y'(0) = 2,$$

where $\delta_c(t)$ is an impulse function (also written $\delta(t - c)$), and A and c are POSITIVE constants.

Find values of A and c so that $y(t) = 0$ for all $t > c$.

Answer: $A =$ _____ $c =$ _____