Math 307A, Midterm 2
Spring 2013

Name: _____

Instructions.

- **DON'T PANIC!** If you get stuck, take a deep breath and go on to the next question. Come back to the question you left if you have time at the end.

- There are 4 questions on 5 pages. Make sure your exam is complete.

- You are allowed one double-sided sheet of notes in your own handwriting. You may not use someone else's note sheet.

- You may use a simple scientific calculator, but you don't need to. No fancy calculators or other electronic devices allowed. If you didn't bring a simple calculator, then just don't use a calculator.

- It's fine to leave your answers in exact form. If you use a calculator, approximate to two decimal places.

- **Show your work**, unless instructed otherwise. If you need more space, raise your hand and I'll give you some extra paper to staple onto the back of your test.

- Don't cheat. If I see that you aren't following the rules, I will report you to UW.

Question	Points	Score
1	16	
2	11	
3	9	
4	10	
Total:	46	

1. (a) (7 points) Find the general solution of the differential equation

$$y'' - 5y' + 2y = 2te^t.$$

(b) (1 point) True or false: **every single solution** to the differential equation in part (a) has the form you found above.

        True   False

(c) (8 points) Find a fundamental set of solutions to the differential equation

$$xy'' + (2 + 2x)y' + 2y = 0.$$

Here is one solution to get you started: $y_1 = x^{-1} = \frac{1}{x}$. You don't need to check that y_1 is a solution.

Clearly label the fundamental set of solutions that is your answer.

2. In this question, use $g = 9.8$ m/s^2 for gravitational acceleration.

   (a) (8 points) A 2-kilogram mass stretches a spring $\frac{g}{2} = 4.9$ meters to its equilibrium position. Marie lifts the mass up one meter and drops it at time $t = 0$. The spring has a damping force F_d of -1 N when the mass has a velocity of 1 m/s. Find a formula for the position u of the mass at time t.

   (b) (2 points) Find a time t_1 so that for all $t \geq t_1$, the mass is within 0.1 units of equilibrium. (In other words, $|u(t)| \leq 0.1$ for every $t \geq t_1$.)

   (c) (1 point) What would γ have to be for this system to be critically damped? (Keep m and k the same; only change γ.)

3. Consider the five graphs below, labeled A–E.
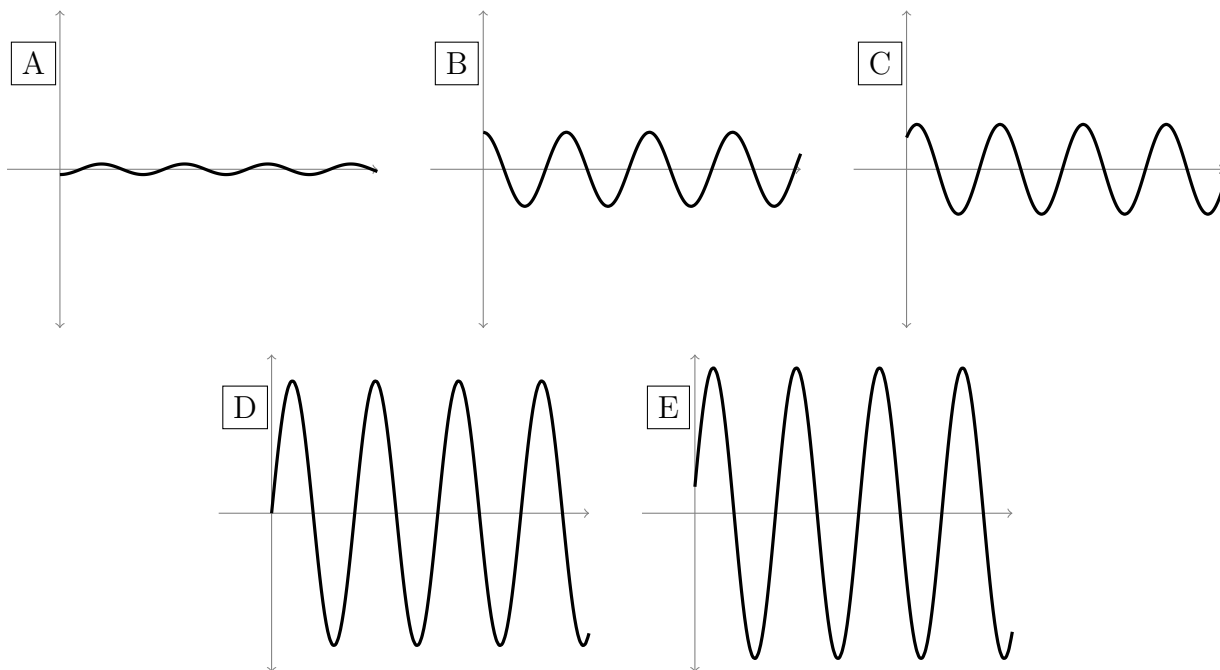
(a) (5 points) Match each differential equation below to the graph **of its steady state.**
Write the graph's label in the space. Each graph matches exactly one equation. You
don't actually need to solve the differential equations to do this problem.

The left side is always the same in the DEs below; only the right side changes. Notice
that γ is small, so there will be a large resonance effect for certain frequences.

$$u'' + 0.01u' + u = \cos(0.09t) \underline{\hspace{2cm}}$$

$$u'' + 0.01u' + u = \cos(0.5t) \underline{\hspace{2cm}}$$

$$u'' + 0.01u' + u = \cos(0.99t) \underline{\hspace{2cm}}$$

$$u'' + 0.01u' + u = \cos(t) \underline{\hspace{2cm}}$$

$$u'' + 0.01u' + u = \cos(9.99t) \underline{\hspace{2cm}}$$

(b) (4 points) In a few complete sentences, explain your reasoning. Be specific. Con-
vince me that you know your answers are correct, and why.

4. (10 points) The motion of a mass on a spring is modeled by the initial value problem

$$u'' + 2u' + 2u = 2\cos(t), \qquad u(0) = 0, u'(0) = 0.$$

Find a formula for **the steady state part** of the solution to this IVP. Put it in the standard form $R\cos(t - \delta)$.