Name: _____

Mathematics 307 Exam

14 February 2001

Instructions: This is a closed book exam: no notes, calculators, or cell phones allowed. It's a good idea to put a box around each solution. Real answers are preferred to complex ones, when there is a choice. The method of undetermined coefficients is preferred to variation of parameters, when there is a choice.

1. (a) (8 points) Find the general solution of $y'' + 4y' + \frac{25}{4}y = 0$.

   (b) (12 points) Find the general solution of $y'' + 3y' = 2$.

2. (5 points) State Euler's formula.

3. Here is a nonhomogeneous differential equation:

$$y'' + 2y' + y = g(t).$$

(a) (6 points) What is y_h, the solution of the associated homogeneous equation?

For the remaining parts, I'll tell you $g(t)$, and I want you to tell me what to try for y_p, according to the method of undetermined coefficients. You don't have to solve for the coefficients, just tell me the right form. For full credit, take your answer for part (a) into account.

(b) (5 points) If $g(t) = 7\sin 4t$, what should you try for y_p?

(c) (5 points) If $g(t) = 6e^{-t}$, what should you try for y_p?

(d) (7 points) If $g(t) = -3e^{2t} + t^3$, what should you try for y_p?

(e) (7 points) If $g(t) = 2t^2e^t \cos 3t$, what should you try for y_p?

4. (a) (7 points) Draw a rough sketch of the function $y(t) = \sin(t)\sin(8t)$.

(b) (8 points) What are the differences between the graphs of the two functions

$$y_1(t) = \cos(t),$$
$$y_2(t) = 4\cos(3t - 1)?$$

5. (10 points) $y_1 = t$ is one solution of the differential equation $t^2y'' + 2ty' - 2y = 0$. Find the general solution.

6. (a) (10 points) Here is the equation of an undamped mass-spring system with an external force acting on it: $3u'' + 6u = 5\cos\omega t$. For what value(s) of ω will the system exhibit resonance?

   (b) (10 points) Here is the equation of a damped mass-spring system with no external force acting on it: $2u'' + \gamma u + 3u = 0$. For what value(s) of γ will the mass exhibit oscillations?

7. (**Bonus**) Find the general solution of the differential equation $y''' - y = t^3$.

Formulas

Trig identities:

$$A\cos\mu t + B\sin\mu t = R\cos(\mu t - \delta),$$
$$\text{where } R = \sqrt{A^2+B^2},\ \delta = \tan^{-1}(B/A);$$
$$\cos\omega t - \cos\omega_0 t = 2\sin\left(\frac{\omega_0-\omega}{2}t\right)\sin\left(\frac{\omega_0+\omega}{2}t\right).$$

Integrals:

$$\int u\,dv = uv - \int v\,du,$$
$$\int \tan u\,du = \ln|\sec u|, \quad \int \cot u\,du = \ln|\sin u|,$$
$$\int \sec u\,du = \ln|\sec u + \tan u|, \quad \int \csc u\,du = \ln|\csc u - \cot u|.$$

Variation of parameters: given the differential equation $y'' + p(t)y' + q(t)y = g(t)$ for which the associated homogeneous equation has linearly independent solutions y_1 and y_2, then

$$W = y_1y_2' - y_1'y_2,$$
$$y_p = -y_1\int \frac{y_2g(t)}{W}dt + y_2\int \frac{y_1g(t)}{W}dt.$$