Your Name

Student ID #

- This exam is closed books. No aids are allowed for this exam. You can use any information on the note sheet without justification.
- In order to receive credit, you must **show all of your work**. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct.
- Give your answers in exact form (for example $\frac{\pi}{3}$ or $5\sqrt{3}$).
- If you need more room, use the backs of the pages and indicate that you have done so.
- Raise your hand if you have a question.
- This exam has 9 pages, plus this cover sheet. Please make sure that your exam is complete.
- You have 1 hour and 50 minutes to complete the exam.

Question	Points	Score
1	7	
2	6	
3	5	
4	7	
5	5	
Total	30	

1. (7 total points)

   (a) (4 points) A tank is filled with 100 gal of solution which initially contains 50 g of sugar. Solution enters the tank at a rate of 200 gal/h. Said solution contains e^{-t} g/gal of sugar after t hours. The well-stirred solution leaves the tank at 200 gal/h. Use the **Laplace transform** to determine the amount of sugar in the tank after any time $t > 0$.

(b) (3 points) Suppose the solution flowing out of the tank mentioned in part (a) flows into a second tank. Initially, the second tank contains 50 gal of solution with an unknown initial amount of sugar, and the solution leaves the tank at a rate of 200 gal/h. Use a method of your choice to determine the amount of sugar in this second tank at any time $t > 0$. *Note: Your answer will contain an unspecified constant.*

2. (6 points) A mass weighing 1 lb stretches a spring 2 ft. The mass is in a perfect vacuum so there is no air resistance. Suppose that an external force of $\frac{1}{16}\cos(4t)$ lb is applied to the mass after t seconds. Finally, suppose that the mass is displaced by 2 ft upward and then set in a motion with a speed of 1 ft/s in the downward direction. Use the **tools from chapter 3** in the textbook (solution formula, method of undetermined coefficients) to determine the position of the mass at any time $t > 0$.

space for problem 2

3. (5 total points) An electrical circuit contains a resistor of $1\,\Omega$ and a capacitor. Its capacitance is varied over time and equals $(1+\cos(t))^{-1}$ F after t seconds. A generator impresses a voltage of $E(t) = e^{-\sin(t)}$ V after t seconds. Assume the initial charge in the circuit is $5\,C$.

   (a) (4 points) Determine the charge at any time $t > 0$.

   (b) (1 point) Determine the limit as $t \to \infty$ of the ratio of the charge and the impressed voltage.

4. (7 points) An electrical circuit is described by the equation

$$Q'' + Q = E$$

where Q is the charge and E is the impressed voltage. Suppose that the circuit is initially closed, i e. $Q(0) = Q'(0) = 0$, and that a generator impressing a voltage of $E = 5e^{-t}\sin(t)$ V is connected to the circuit at time $t = \pi$. Use the **Laplace transform** to determine the charge at any time $t > 0$. *Hint: All the identities you need are on the note sheet.*

space for problem 4

5. (5 total points)

   (a) (3 points) A ball with mass m is dropped (i. e. with zero initial velocity) from the roof of a building. Assume that the force due to air resistance is proportional to the square of its velocity where the velocity is measured in m/s. Determine the velocity of the ball for all sufficiently small times $t > 0$ (i. e. you do not need to worry about what happens after the ball hits the ground). You need to mention if you take the downward direction to be the positive or negative direction; either choice is fine.

(b) (1 point) Assume the situation in part (a) except that the ball is dropped from an airplane. Since the ball is dropped from a considerable height, we need to assume that the resistance force depends not only on air resistance, but also on air pressure. Assume that the resistance is proportional to the product of air resistance and air pressure. Also, assume that the air pressure varies exponentially with the altitude. You can assume that the force by gravity is independent of the altitude of the ball. Set up a differential equation which describes the motion. *Note: A quantity x varies exponentially if it is proportional to e^{-ax} for some $a > 0$.*

(c) (1 point) Show that the velocity of the object satisfies a first-order differential equation. *Hint: Proceed to solve the differential equation you set up in (b). The result will pop out mid-way.*