1. (0 total points)

   (a) (5 points) Find the general solution to the following second-order differential equation:

   $$3y'' + 2y' - y = 4e^{-t}\cos(t) + 2e^{-t}.$$

   (b) (5 points) Find the solution $y = \phi(t)$ to the following initial value problem.

   $$4y'' + y = 2\cos(t) \qquad y(0) = 0,\ y'(0) = 1.$$

2. (0 total points) Consider the initial value problem

$$(\alpha - 2)y'' + (3\alpha)y' + (2\alpha + 1)y = 0, \qquad y(0) = 1, y'(0) = 0$$

for a given constant α.

(a) (5 points) Find the values of α for which the solution to the IVP exhibits oscillatory behavior. For these values will the solution's oscillations be damped, constant in amplitude or exponentially growing?

(b) (5 points) Let α be the value which maximizes the solution's quasi-frequency, and let $y(t)$ be the solution to the IVP for this value of α. Find a time t_0 beyond which the amplitude of y never exceeds 0.1, i.e. for which $|y(t)| \leq 0.1$ for all $t > t_0$.

3. (0 total points) A certain vibrating system satisfies the differential equation

$$0.5y'' + 0.1y' + 2y = 3\cos(\omega_0 t)$$

where ω_0 is the natural frequency of the system.

(a) (5 points) Compute the amplitude of the system's steady-state solution.

(b) (5 points) Suppose the forcing function's frequency is doubled to $2\omega_0$, but everything else remains the same. What does the amplitude of the steady-state solution now become?

4. (0 total points) A series circuit contains a capacitor of 6.4×10^{-4} F and an inductor of 10 H. Resistance in the circuit is negligible, and the charge on the capacitor and the current in the circuit are both initially zero. At time $t = 0$ an external voltage is applied to the circuit of $125\cos(15t)$ volts.

   (a) (6 points) Formulate and solve an initial value problem using the above data to determine the charge on the capacitor at time t.

   (b) (4 points) The capacitor is rated to sustain a maximum charge of 0.5 Coulombs. Is this circuit safe given the above setup, or will it burn out?

5. (10 points) (0 total points) An object of unknown mass is placed on a flat surface and attached to a horizontal spring with spring constant 2.5 kg/s^2. The damping constant in the system is precisely 1 kg/s. The object is stretched 1 meter to the right of its equilibrium position and released with zero initial velocity. The damped oscillations of its subsequent motion are observed to have a quasi-period of $\frac{20}{7}\pi$ seconds.

   What is the mass of the object?