Your Name

Student ID #

- Cellphones off please!
- You are allowed one two-sided handwritten notesheet for this midterm. You may use a scientific calculator; graphing calculators and all other course-related materials may not be used.
- In order to receive credit, you must **show all of your work**. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct.
- Give your answers in exact form (for example $\frac{\pi}{3}$ or $e^{-5\sqrt{3}}$) unless explicity stated otherwise by the question.
- You may quote and use any formula you have seen in class to save time, but be sure to indicate with a word or to that you are doing so.
- If you need more room, use the backs of the pages and indicate that you have done so.
- Raise your hand if you have a question.
- This exam has 6 pages, plus this cover sheet. Please make sure that your exam is complete.
- You have 50 minutes to complete the exam.

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total	50	

1. (10 points) Find the general solution to the following second-order differential equation:

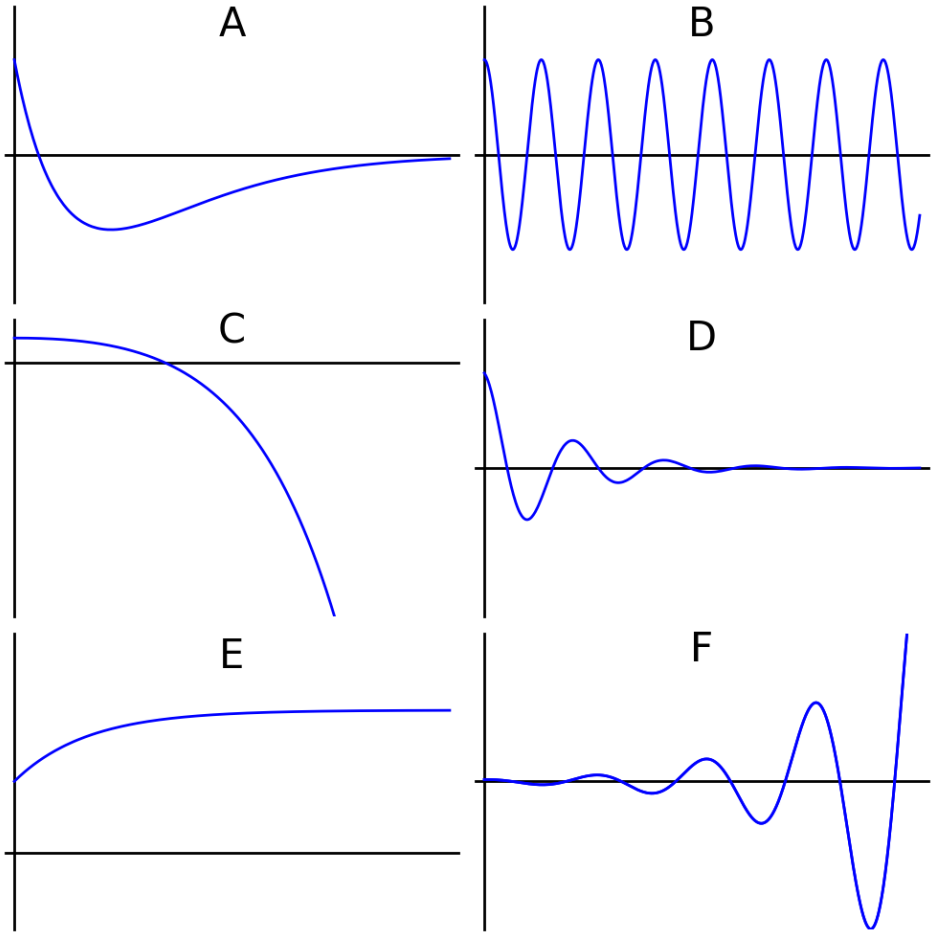
$$4y'' - 12y' + 9y = 9t.$$

2. (10 points) Consider the differential equation

$$t^2y'' - 4ty' + 6y = 0.$$

One can check that $y_1(t) = t^2$ obeys this DE. Use the method of reduction of order or any other method of your choosing to find the solution subject to the initial conditions $y(1) = 1$, $y'(1) = 0$.

3. (10 points) Below are the graphs of six functions $y(t)$, with t and y being the horizontal and vertical axes respectively. The graphs are labeled A through F. The graphs are **not** all drawn to the same scale, and axis markings have been purposely omitted.



Each of the functions graphed above is a solution to exactly one of the six differential equations below. By analyzing the form of the equations' general solutions, write the letter of the graph next to the differential equation for which it is the solution. You do not need to show your work in this question to receive full credit.

$$y'' - 3y' + 2y = 0 \quad \_\_\_\_\_$$

$$y'' + 16y = 0 \quad \_\_\_\_\_$$

$$y'' - y' + \frac{3}{2}y = 0 \quad \_\_\_\_\_$$

$$y'' + y' + \frac{3}{2}y = 0 \quad \_\_\_\_\_$$

$$y'' + y' + \frac{1}{4}y = 0 \quad \_\_\_\_\_$$

$$y'' + 2y' = 0 \quad \_\_\_\_\_$$

4. (10 total points) A series circuit contains a capacitor of 2×10^{-4} F, an inductor of 2 H, and a resistor of R ohms. Consider the differential equation governing the charge $Q(t)$ on the capacitor as a function of time, where Q is in Coulombs and t in seconds.

   (a) (2 points) We ascertain that R is such that the system exhibits critical damping. Find R.

   (b) (6 points) The initial charge on the capacitor is zero and there is no initial current. Starting at time $t = 0$, a constant external voltage of 100 volts is applied, where t is in seconds. Find the the charge on the capacitor as a function of time. What is the amplitude of the steady-state response?

   (c) (2 points) The external voltage is now changed to $100\cos(50t)$ volts. Will the amplitude of the steady-state response increase, decrease or stay the same compared to what you found in part (b)? Justify your answer.

5. (10 total points + 3 bonus points) A $\frac{1}{4}$ kg mass is placed on a flat frictionless surface and attached to a horizontal spring. It takes 4 N of force to move the mass 36 cm to the right of its equilibrium position. The mass starts at rest in its equilibrium position. Starting at time $t = 0$ seconds a horizontal force of $0.41\cos(7t)$ Newtons acts on the mass. Friction in this problem is negligible.

   (a) (3 points) Formulate an initial value problem that describes the position of the mass at time t.

   (b) (5 points) Solve the above initial value problem to find the position of the mass at time t. You may use known formulae to save time, but be sure to indicate if you are quoting a formula you've seen in class.

[NB: Question continued on the next page!]

(c) (2 points) What is the maximum distance the mass achieves from its equilibrium position?

(d) (Bonus: 3 points) Estimate the maximum amount of kinetic energy that the mass will have.