

Math 310 Assignment 5

PROBLEMS: 4.43, 4.45, 4.47, 6.1, 6.2, 6.3, 6.4, 6.8

In addition, complete the following 2 supplemental problems:

PROBLEM I: Let $f(x) = \begin{cases} 2x + 1 & , \text{ if } x \geq 0; \\ -2x & , \text{ if } x < 0. \end{cases}$

Prove that \mathbb{Z} is countable by showing that $f : \mathbb{Z} \rightarrow \mathbb{N}$ is a bijection.

PROBLEM II: Let A be the set of rational numbers and let B be the set of irrational numbers. Are A and B both countable? If so, why? If not, which has a larger cardinality? (Give a short justification using the results from class).

The problems above are DUE FRIDAY, November 9th at lecture or during office hours.

HOMEWORK NOTES/HINTS

- I challenge each of you to start the homework early and visit my office hours on Monday and Wednesday next week to discuss the problems.
- **Problem 4.43:** There is a typo, it should say “Let B be a proper subset of A , let f be a bijection from A to B , and let B be an infinite set. Prove that A is an infinite set.”
You can use exercise 4.42 without proving it. You might want to try a proof by contradiction.
- **Problem 4.47:** You need to define two functions (one from the even natural numbers to the natural numbers and the other from the odd natural numbers to the natural numbers) and you need to prove that your functions are bijections (by showing each is one-to-one and onto).
- **Problems 6.1, 6.2, 6.3, 6.8:** These are all short answer problems. That is, you do not need to give proofs. However, you should still show your work so that I know how to get the answer.
- If you finish the homework early or if you are looking for some extra practice try the following problems:

CHALLENGE PROBLEMS: 4.48, 6.51, 6.52

These are not due, but if you complete them all correctly and hand them in with your homework, I will award at least 1/2 a point of extra credit per challenge problem completed.