Integral Application List

$\mathbb{R}^1$ Applications:

$$\int_a^b 1\,dx = b - a = \text{length of interval.}$$

$$\frac{1}{b-a}\int_a^b f(x)\,dx = \text{average value of } f(x) \text{ over } [a,b].$$

$$\int_a^b f(x)\,dx = \text{net area of } f(x) \text{ over } [a,b].$$

$$\int_a^b \sqrt{(f'(x))^2+1}\,dx = \text{arc length of } f(x) \text{ over } [a,b].$$

$\mathbb{R}^2$ Applications:

$$\iint_D 1\ dA = \text{area of } D.$$

$$\frac{1}{\text{Area of } D}\iint_D f(x,y)\ dA = \text{average value of } f(x,y) \text{ over } D.$$

$$\iint_D f(x,y)\ dA = \text{net volume of } f(x,y) \text{ over } D.$$

$$\iint_D \sqrt{(f_x(x,y))^2+(f_y(x,y))^2+1}\ dA = \text{surface area of } f(x,y) \text{ over } D.$$

$$Q = \iint_D \sigma(x,y)\ dA = \text{total charge over } D.$$

$$M = \iint_D \rho(x,y)\ dA = \text{total mass.}$$

$$M_y = \iint_D x\rho(x,y)\ dA = \text{moment } \textit{about} \text{ the } y\text{-axis, and} \quad \bar{x} = \frac{M_y}{M}.$$

$$M_x = \iint_D y\rho(x,y)\ dA = \text{moment } \textit{about} \text{ the } y\text{-axis, and} \quad \bar{y} = \frac{M_x}{M}.$$

$$I_y = \iint_D x^2\rho(x,y)\ dA = \text{moment of inertia } \textit{about} \text{ the } y\text{-axis.}$$

$$I_x = \iint_D y^2\rho(x,y)\ dA = \text{moment of inertia } \textit{about} \text{ the } x\text{-axis.}$$

$$I_o = \iint_D (x^2+y^2)\rho(x,y)\ dA = I_y + I_x = \text{moment of inertia } \textit{about} \text{ the origin.}$$

$\mathbb{R}^3$ Applications:

$$\iiint_E 1 \; dV = \text{volume of } E.$$

$$\frac{1}{\text{Volume of } E} \iiint_E f(x,y,z) \; dV = \text{average value of } f(x,y,z) \text{ over } E.$$

$$\iiint_E f(x,y,z) \; dV = \text{hypervolume of } f(x,y,z) \text{ over } E \text{ (No visualization).}$$

$$Q = \iiint_E \sigma(x,y,z) \; dV = \text{total charge over } E.$$

$$M = \iiint_E \rho(x,y,z) \; dV = \text{total mass.}$$

$$M_{yz} = \iiint_E x\rho(x,y,z) \; dV = \text{moment } \textit{about} \text{ the } yz\text{-plane, and } \quad \bar{x} = \frac{M_{yz}}{M}.$$

$$M_{xz} = \iiint_E y\rho(x,y,z) \; dV = \text{moment } \textit{about} \text{ the } xz\text{-plane, and } \quad \bar{y} = \frac{M_{xz}}{M}.$$

$$M_{xy} = \iiint_E z\rho(x,y,z) \; dV = \text{moment } \textit{about} \text{ the } xy\text{-plane, and } \quad \bar{z} = \frac{M_{xy}}{M}.$$

$$I_z = \iiint_E (x^2+y^2)\rho(x,y,z) \; dV = \text{moment of inertia } \textit{about} \text{ the } z\text{-axis}$$

$$I_y = \iiint_E (x^2+z^2)\rho(x,y,z) \; dV = \text{moment of inertia } \textit{about} \text{ the } y\text{-axis}$$

$$I_x = \iiint_E (y^2+z^2)\rho(x,y,z) \; dV = \text{moment of inertia } \textit{about} \text{ the } x\text{-axis}$$

$$I_o = \iiint_E (x^2+y^2+z^2)\rho(x,y,z) \; dV = \tfrac{1}{2}(I_x+I_y+I_z) = \text{moment of inertia } \textit{about} \text{ the origin.}$$