LINE AND SURFACE INTEGRAL FLOW CHARTS

The charts on the next two page give one summary of how to think about computing line and surface integrals given various situations.

Given a curve, **C**, parameterized by **r**(t), $a \le t \le b$

Integrating a scalar function with respect to arc length:

z=f(x,y) or **w=f(x,y,z)**

- **Use parameterization**
  Replace
  $x = x(t)$, $y = y(t)$, $z = z(t)$, and
  $ds = |\mathbf{r}'(t)|dt$

Integrating over a vector field:

$\mathbf{F}(x,y) = P(x,y,z)\mathbf{i} + Q(x,y,z)\mathbf{j}$

$\mathbf{F}(x,y,z) = P(x,y,z)\mathbf{i} + Q(x,y,z)\mathbf{j} + R(x,y,z)\mathbf{k}$

- **Conservative**
  Only if curl $\mathbf{F} = \mathbf{0}$
  In 2D, that condition becomes: $P_y = Q_x$
  - **Use the paramterization**
    Replace $x = x(t)$, $y = y(t)$, $z = z(t)$, $d\mathbf{r} = \mathbf{r}'(t)dt$
  - **Find a potential function.**
    1. Integrate P wrt x
    2. Integrate Q wrt y
    3. Integrate R wrt z
    4. Find f.
- **Non conservative**
  curl $\mathbf{F} \neq \mathbf{0}$, or in 2D, $P_y \neq Q_x$
  - **Use the paramterization**
    Replace $x = x(t)$, $y = y(t)$, $z = z(t)$, $d\mathbf{r} = \mathbf{r}'(t)dt$
  - **Closed curve**
    Use
    Green's Theorem or Stoke's Theorem

Given a surface, **S**, parameterized by **r**(u,v), where (u,v) come from some domain D.

Integrating a scalar function with respect to surface area:

w=f(x,y,z)

Use param.

Replace

x = x(u,v),

y = y(u,v),

z = z(u,v)

dS = |$\mathbf{r}_u \times \mathbf{r}_v$| dA

Integrating over a vector field:

F(x,y,z) **=** P(x,y,z)**i** + Q(x,y,z)**j** + R(x,y,z)**k**

If div **F = 0,**

then **F** can be written as the curl of another vector field, so

F = curl **G**

Use the paramterization

Replace x = x(u,v), y = y(u,v), z = z(u,v), d**S** = ($\mathbf{r}_u \times \mathbf{r}_v$)dA

If you can find **G**, then try Stoke's Theorem.

If div **F** ≠ **0**,

then **F** cannot be written as the curl of another vector field.

Use the paramterization

Replace x = x(u,v), y = y(u,v), z = z(u,v), d**S** = ($\mathbf{r}_u \times \mathbf{r}_v$)dA

Closed surface

Use the Diverence Theorem