Integrals Summary

Double integrals over regions in $\mathbf{R}^2$: $$\iint_D f(x,y)\,dA.$$

1. Find the top and bottom inner bound equations: $g_1(x) \le y \le g_2(x)$ and constant outer bounds: $a \le x \le b$.
2. Find the right and left inner bound equations: $h_1(y) \le x \le h_2(y)$ and constant outer bounds: $c \le y \le d$.

Triple integrals over solids in $\mathbf{R}^3$: $$\iiint_E f(x,y,z)\,dV.$$

1. Find the top and bottom inner bound equations: $g_1(x,y) \le y \le g_2(x,y)$.
   Draw xy-plane projection of solid to find outer bounds for x and y.
2. Find the right and left inner bound equations: $g_1(x,z) \le y \le g_2(x,z)$.
   Draw xz-plane projection of solid to find outer bounds for x and z.
3. Find the front and back inner bound equations: $g_1(y,z) \le x \le g_2(y,z)$.
   Draw yz-plane projection of solid to find outer bounds for y and z.

Line integrals for scalar functions: $$\int_C f(x,y,z)\,ds = \int_a^b f(\mathbf{r}(t))|\mathbf{r}'(t)|\,dt.$$

1. Parameterize: $\mathbf{r}(t) = \langle x(t), y(t), z(t)\rangle$ with $a \le t \le b$.
2. Compute: $|\mathbf{r}'(t)| = \sqrt{(x'(t))^2 + (y'(t))^2 + (z'(t))^2}$.
3. Replace: $x = x(t)$, $y = y(t)$ and $z = z(t)$ to get $f(\mathbf{r}(t)) = f(x(t), y(t), z(t))$. Integrate!

Line integrals for vector functions: $$\int_C \mathbf{F}\cdot d\mathbf{r} = \int_a^b \mathbf{F}(\mathbf{r}(t))\cdot\mathbf{r}'(t)\,dt.$$

1. Parameterize: $\mathbf{r}(t) = \langle x(t), y(t), z(t)\rangle$ with $a \le t \le b$.
2. Compute: $\mathbf{r}'(t) = \langle x'(t), y'(t), z'(t)\rangle$.
3. Replace: $x = x(t)$, $y = y(t)$ and $z = z(t)$ to get $\mathbf{F}(\mathbf{r}(t)) = \langle P(\mathbf{r}(t)), Q(\mathbf{r}(t)), R(\mathbf{r}(t))\rangle$. Integrate! Also, check the orientation!

Surface Integrals for scalar functions: $$\iint_S f(x,y,z)\,dS = \iint_D f(\mathbf{r}(t))|\mathbf{r}_u \times \mathbf{r}_v|\,dA.$$

1. Parameterize: $\mathbf{r}(u,v) = \langle x(u,v), y(u,v), z(u,v)\rangle$.
2. Compute: $|\mathbf{r}_u \times \mathbf{r}_v|$.
3. Replace: $x = x(t)$, $y = y(t)$ and $z = z(t)$ to get $f(\mathbf{r}(t)) = f(x(t), y(t), z(t))$.
4. Find domain for your parameters u and v. Integrate!

Surface Integrals for vector functions: $$\iint_S \mathbf{F}\cdot d\mathbf{r} = \iint_D \mathbf{F}(\mathbf{r}(u,v))\cdot(\mathbf{r}_u \times \mathbf{r}_v)\,dA.$$

1. Parameterize: $\mathbf{r}(u,v) = \langle x(u,v), y(u,v), z(u,v)\rangle$.
2. Compute: $\mathbf{r}_u \times \mathbf{r}_v$.
3. Replace: $x = x(t)$, $y = y(t)$ and $z = z(t)$ to get $\mathbf{F}(\mathbf{r}(u,v)) = \langle P(\mathbf{r}(u,v)), Q(\mathbf{r}(u,v)), R(\mathbf{r}(u,v))\rangle$.
4. Find domain for your parameters u and v. Integrate! Also, check the orientation!