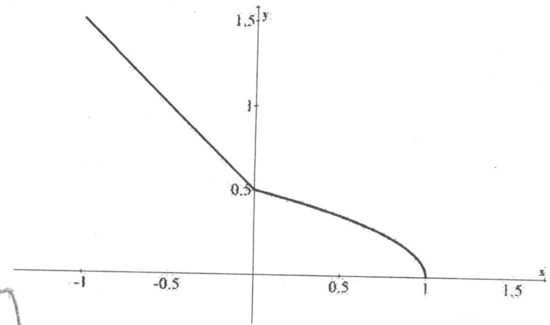
NAME: SOLUTION

Test Prep 2

Let C_1 be the arc of the curve $x + 4y^2 = 1$ from $(1, 0)$ to $(0, \frac{1}{2})$.
And let C_2 be the line segment from $(0, \frac{1}{2})$ to $(-1, \frac{3}{2})$.
Let C consist of C_1 followed by C_2. The curve C is shown below with the desired orientation.



1. Give a parameterization for C_1.

$x = 1 - 4y^2$

$$\boxed{x = 1 - 4t^2 \qquad y = t \qquad 0 \le t \le \tfrac{1}{2}}$$

Check orientation: $t = 0 \Rightarrow (1, 0)$ ✓
$t = \frac{1}{2} \Rightarrow (0, \frac{1}{2})$

2. Give a parameterization for C_2.

$$\left.\begin{aligned} x &= (x_1 - x_0)t + x_0 \\ y &= (y_1 - y_0)t + y_0 \\ 0 &\le t \le 1 \end{aligned}\right\}$$

$$\boxed{x = -t + 0 \qquad y = t + \tfrac{1}{2} \qquad 0 \le t \le 1}$$

check: $t = 0 \Rightarrow (0, \frac{1}{2})$
$t = 1 \Rightarrow (-1, \frac{3}{2})$

3. Let $\mathbf{F} = \langle x, -y \rangle$ be a vector field.
Using your parameterizations, compute $\displaystyle\int_C \mathbf{F} \cdot d\mathbf{r} = \int_{C_1} \mathbf{F} \cdot d\mathbf{r} + \int_{C_2} \mathbf{F} \cdot d\mathbf{r}$

$$\int_{C_1} \vec{F} \cdot d\vec{r} = \int_{C_1} x\,dx - y\,dy = \int_0^{1/2} (1 - 4t^2)(-8t)\,dt - t(1)\,dt$$

$$= \int_0^{1/2} -8t + 32t^3 - t\,dt = \int_0^{1/2} -9t + 32t^3\,dt$$

$$= -\frac{9}{2}t^2 + 8t^4 \Big|_0^{1/2} = -\frac{9}{8} + \frac{8}{16} = -\frac{9}{8} + \frac{4}{8} = -5/8$$

$$\int_{C_2} \vec{F} \cdot d\vec{r} = \int_0^1 (-t)(-1)\,dt - (t + \tfrac{1}{2})(1)\,dt$$

$$= \int_0^1 t - t - \tfrac{1}{2}\,dt = -\tfrac{1}{2}t\Big|_0^1 = -\tfrac{1}{2}$$

$$\int_C \vec{F} \cdot d\vec{r} = -5/8 - 1/2 = -\frac{5}{8} - \frac{4}{8} = \boxed{-9/8}$$