Time allowed: 1 hour and 50 minutes. Total points = 100.
Only nongraphing calculators allowed. You may use one 8.5" x 11" sheet of handwritten notes.
Show all your work in a clear and organized fashion so that I can follow your reasoning. Include some English words as well as symbols and equations! *A correct final answer that is not supported by reasoning and calculations may not receive full credit.*
Give "exact answers", that is, in terms of π, $\sqrt{2}$, etc.
If you use a major theorem (Green's, Stokes, Divergence), say so!
Raise your hand if you have a question or need more paper.

ON AN ACTUAL FINAL, SPACE WILL BE PROVIDED FOR WORKING THE PROBLEMS.

1. (15 points) a) Indicate by circling the appropriate word whether each expression is a **scalar function**, a **vector** field, or a **nonsense** expression.

Suppose $f(x, y, z)$ is a scalar function and $\mathbf{F}(x, y, z)$ and $\mathbf{G}(x, y, z)$ are vector fields. Assume all three are "nice;" that is, they are continuous and have continuous partial derivatives of all orders.

$div(grad\, f)$:	**scalar function**	**vector**	**nonsense**
$curl(\nabla f)$:	**scalar function**	**vector**	**nonsense**
$curl(\nabla \cdot \mathbf{F})$:	**scalar function**	**vector**	**nonsense**
$(\nabla \mathbf{F}) \cdot \mathbf{G}$:	**scalar function**	**vector**	**nonsense**
$(\nabla \cdot \mathbf{F})(\nabla \times \mathbf{G})$:	**scalar function**	**vector**	**nonsense**
$\lvert curl(\mathbf{F}) \times \mathbf{G} \rvert$:	**scalar function**	**vector**	**nonsense**

b) Which expressions above, if any, are always zero (no matter what the function and the vector fields are)? (Note: "zero" can mean the scalar 0 or the vector $\vec{0}$.) Write "NONE" if there are none, and list all answers if there are more than one.

2. (15 points) Let C be the curve consisting of a straight line segment from the origin to $(2, 0)$, then one quarter of the circle $x^2 + y^2 = 4$ from $(2, 0)$ to $(0, -2)$.

(a) Compute $\int_C x\, ds$.

(b) Compute $\int_C x\, dy$.

3. (13 points) Let E be region bounded below by the plane $z = -5$ and above by the $z \leq 0$ part of $x^2 + y^2 = z^2$. (So E is the solid cone with circular base on the plane $z = -5$.)

(a) Set up the integral $\iiint_E (e^x + z^2)\, dV$ in Cartesian $(x,\ y,\ z)$ coordinates.

(b) Set up the integral $\iiint_E (e^x + z^2)\, dV$ in your choice of cylindrical or spherical coordinates. (*The limits and integrand should be in terms of your chosen coordinates only, not Cartesian coordinates.*)

4. (18 points) Let S be the portion of the generalized cylinder $x^2 + 4z^2 = 16$ between $y = 0$ and $y = 10$, oriented by the outward (away from the y-axis) normal.

(a) Give a parametrization $\mathbf{r}(u, v)$ of S, including specifying the domain (that is, the bounds on (u, v)). Does $\mathbf{r}_u \times \mathbf{r}_v$ give the orientation specified, or the opposite orientation? Explain briefly.

(b) The boundary of S comes in two pieces, C_1 in the $y = 0$ plane and C_2 in the $y = 10$ plane.

(i) Give a parametrization of C_1. Does your parametrization give the orientation of C_1 consistent with the given orientation of S, or the opposite orientation? Explain briefly.

(ii) Give a parametrization of C_2. Does your parametrization give the orientation of C_2 consistent with the given orientation of S, or the opposite orientation? Explain briefly.

5. (15 points) Let S be the part of the surface $x = 1 - y^2 - z^2$ where $x \geq 0$, $z \geq 0$, and $0 \leq y \leq z$, oriented towards the origin.

Compute $\iint_S \mathbf{F} \cdot d\mathbf{S}$, where $\mathbf{F} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$. Be sure to explain your reasoning about orientation.

6. (12 points) Let C be the closed curve consisting of line segments starting at the origin going to $(0, 3, 0)$, then to $(1, 0, 1)$, then back to the origin. Let $\mathbf{F} = \langle 2x - 3z, y + 7x, 5y - z, \rangle$. Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ (You may compute it directly, or use one of the theorems of chapter 16. It may be useful to note that C is a right triangle.)

7. (12 points) For both parts of this problem, let S_1 be the upper ($z \geq 0$) half of the sphere $x^2 + y^2 + z^2 = 9$ and S_2 the disk $x^2 + y^2 \leq 9$ in the xy-plane, both surfaces oriented by the upward normal. Let E be the region between S_1 and S_2; that is, E is the region where $x^2 + y^2 + z^2 \leq 9$ and $z \geq 0$.

(a) Let $\mathbf{F}$ be any vector field that has continuous partial derivatives on an open set containing E. Explain why the following equation holds:

$$\iint_{S_1} \mathbf{F} \cdot d\mathbf{S} = \iiint_E div(\mathbf{F}) dV + \iint_{S_2} \mathbf{F} \cdot d\mathbf{S}.$$

Be sure to pay attention to orientation issues in your discussion.

(b) Let $\mathbf{F} = \langle e^{yz}, y + x^3 z, x + z \rangle$. Evaluate $\iint_{S_1} \mathbf{F} \cdot d\mathbf{S}$.

EXTRA CREDIT (10 points). Little or no partial credit: do not work on this problem until you have done everything you can with the rest of the test.

Extra credit problem involved reasoning about vector fields from pictures of the fields.