1. Consider two vector fields: $\mathbf{F} = \langle x + z, 1, x\rangle$ and $\mathbf{G} = \langle y, -x, e^z\rangle$.

   a) For each of the two fields, determine whether it is conservative. **Show your reasoning!** Give a potential function for each conservative field.

   b) Let C be the curve from (0,0,0) to (4,2,20) along the intersection of the surfaces defined by $x^2 + y^2 = z$ and $x = 2y$. Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ and $\int_C \mathbf{G} \cdot d\mathbf{r}$.

2. The function g of three variables is given by $g(x, y, z) = xz^2 + y - e^6$.

   (a) Suppose $\mathbf{r}(t)$ is a parametrized curve; we do not know the formulas for $\mathbf{r}(t)$, but we know that $\mathbf{r}(5) = \langle 2, -7, 3\rangle$ and $\mathbf{r}'(5) = \langle -1, \pi, 2\rangle$. Define a new function $h(t) = g(\mathbf{r}(t))$; find $h'(5)$.

   (b) Find the equation of the tangent plane to the level set for g through the point $(2, -7, 3)$.

   (c) Suppose you are at the point (2,-7,3), and you want to start moving in a direction so that g stays constant. Give one possible direction for which this is true.

3. Let S be the part of the surface $y = z^2$ inside the cylinder $x^2 + z^2 = 4$, oriented by the normal with positive $\mathbf{j}$ component.

   (a) Give a parametrization $\mathbf{r}(u, v)$ of S, including specifying the domain (that is, the bounds on (u, v)). Does $\mathbf{r}_u \times \mathbf{r}_v$ give the orientation specified, or the opposite orientation?

   (b) Give a parametrization of the boundary curve C of S as a function of t, including specifying the interval for t. Does your parametrization give the orientation of C consistent with the given orientation of S, or the opposite orientation?

   (c) Compute $\iint_S \nabla \times \mathbf{F} \cdot d\mathbf{S}$, where $\mathbf{F} = z\mathbf{i} + (4 - x^2 - z^2)\mathbf{j} - x\mathbf{k}$. (You may compute it directly, or use one of the theorems of chapter 16.)

4. Let S be part of the cylinder $x^2 + y^2 = 9$ where $0 \le z \le 5$. Let $f(x, y, z) = 2z$, and let $\mathbf{F} = \mathbf{i} + \mathbf{k}$.

   Determine whether each of the following expressions makes sense. If it doesn't make sense, say briefly why. If it does make sense, compute it. (Hint: you may be able to reason directly from the meaning of the surface integrals and compute them without setting up a parametrization.)

   (a) $\iint_S f\, dS$
   (b) $\iint_S f \cdot d\mathbf{S}$
   (c) $\iint_S \mathbf{F} \cdot d\mathbf{S}$

5. Reasoning from pictures of vector fields: p. 1044-1045, #17, 18, 47; p. 1054, #23-24;
   p. 1068, #9-11 (can use ideas from later sections, pp. 1096 and 1103);
   p. 1104, #19-20.