Math 324B
FIRST PRACTICE EXAM
(a bit longer than the real exam)

1. Let D be the region between the line $y = 3x + 4$ and the parabola $y = 4 - x^2$. Express $\iint_D 2x\, dA$ as an iterated integral in two ways ($dy\, dx$ and $dx\, dy$), and evaluate it. (For practice, do it both ways.)

2. Let E be the region in the first octant (x, y, z all > 0) below the plane $3x + 2y + z = 6$. Set up $\iiint_E f(x, y, z)\, dV$ as an iterated integral in the order (i) $dz\, dy\, dx$, (ii) $dy\, dx\, dz$, and (iii) $dx\, dz\, dy$.

3. Let D be the region in the first quadrant of the plane inside the circle $x^2 + y^2 = 1$ and above the line $y = x/\sqrt{3}$. Use polar coordinates to evaluate $\iint_D e^{-x^2-y^2}\, dA$.

4. Use cylindrical coordinates to find the mass of a body that occupies the region between the paraboloid $z = x^2 + y^2$ and the plane $z = 4$ if the mass density is $\rho(x, y, z) = 8 - 2z$.

5. Let E be the region in the first octant inside the sphere of radius 2 about the origin and outside the sphere of radius 1 about the origin.

   a. Convert $\iiint_E f(x, y, z)\, dV$ into an iterated integral (in any order you like) in spherical coordinates.

   b. What is the center of mass $(\overline{x}, \overline{y}, \overline{z})$ of a solid body that occupies the region E if its mass density is identically equal to 1? (Hint: Use geometry. In the first place, since the density is 1, the mass is the volume, and you don't really need integration to compute that. Moreover, $\overline{x} = \overline{y} = \overline{z}$; why?)

6. Let R be the region in the first quadrant of the xy-plane between the lines $x + y = 1$ and $x + y = 3$.

   a. What is the region in the uv-plane corresponding to R under the transformation $x = u - uv$, $y = uv$? (Hint: First verify that $u = x + y$ and $v = y/(x + y)$. What is v when $x = 0$ or $y = 0$?)

   b. Use the transformation in part (a) to calculate $\displaystyle\iint_R \frac{1}{(x + y)^2}\, dA$.