Math 324B
FINAL PRACTICE EXAM

Notes (these pertain to the real exam too): (1) If you use a major theorem (Green, Stokes, etc.), mention it by name! (2) It's OK to use shortcuts in evaluating integrals (e.g., observing that an integral is zero because of some symmetry property) if you briefly explain what you're doing.

1. Let E be the region inside the sphere $x^2+y^2+z^2=2$ and above the cone $z=\sqrt{x^2+y^2}$. Express $\iiint_E e^{xz}\,dV$ in spherical coordinates. You don't have to evaluate it.

2. Express $\displaystyle\int_{-3}^{1}\int_{2x}^{3-x^2} f(x,y)\,dy\,dx$ as an integral (or sum of integrals) with the order of integration reversed. (That is, find the right limits of integration.)

3. Let D be the triangle in the xy-plane with vertices $(0,0)$, $(-1,1)$, and $(-2,0)$. Evaluate $\displaystyle\iint_D \cos\frac{\pi(x+y)}{2(x-y)}\,dA$ with the help of the transformation $u=x+y$, $v=x-y$.

4. Let E be the region in the first octant below the plane $x+2y+3z=6$, let S be its boundary (with the usual outward orientation), and let $\mathbf{F}(x,y,z)=(x^2+\sin\pi y)\mathbf{i}+\sqrt{x^4+z^4+1}\,\mathbf{j}+xz\mathbf{k}$. Evaluate $\displaystyle\iint_S \mathbf{F}\cdot d\mathbf{S}$.

5. Let $\mathbf{F}(x,y,z)=(e^{x/2}+\sin(3y+4z))\mathbf{i}+3x\cos(3y+4z)\mathbf{j}+(4x\cos(3y+4z)-z^2)\mathbf{k}$.
   a. Compute $\operatorname{curl}\mathbf{F}$ and $\operatorname{div}\mathbf{F}$.
   b. Is there a function f such that $\mathbf{F}=\nabla f$? If so, find such an f. If not, explain why.
   c. Is there a vector field $\mathbf{G}$ such that $\mathbf{F}=\operatorname{curl}\mathbf{G}$? If so, find such a $\mathbf{G}$. If not, explain why.

6. Let C_1 be the ellipse $(x/2)^2+(y/3)^2=1$, oriented clockwise, and let C_2 be its right-hand half. Compute $\displaystyle\int_{C_1}(xy\,dx-x^2\,dy)$ and $\displaystyle\int_{C_2}(xy\,dx-x^2\,dy)$. (There's an easy way to do the first one!) Also, set up an ordinary one-dimensional integral that represents $\displaystyle\int_{C_2} x\,ds$; you don't have to evaluate it.

7. Let S be the piece of the hyperboloid $x^2+y^2-z^2=1$ where $0\le z\le 1$, oriented with the normal pointing outward (away from the z-axis). Evaluate $\displaystyle\iint_S(x\mathbf{i}+y\mathbf{j}+z\mathbf{k})\cdot d\mathbf{S}$ and $\displaystyle\iint_S z\,dS$. (Hint: In cylindrical coordinates this surface is given by $r=\sqrt{1+z^2}$.)

8. Let S be the surface parametrized by $\mathbf{r}(u,v)=(2u+v)\mathbf{i}+(4u+3v)\mathbf{j}+u^2\mathbf{k}$ with $0\le u\le 1$, $0\le v\le 1$, oriented upward, and let C be its boundary with the compatible orientation. Evaluate $\displaystyle\int_C \mathbf{F}\cdot d\mathbf{r}$, where $\mathbf{F}(x,y,z)=(2z+\sqrt{x^2+1})\mathbf{i}+(3x+e^{y^2})\mathbf{j}+y\mathbf{k}$.