Assignment 1, due Wednesday, April 8.

Reading: Finish Chapter 6. There will be some notes giving the alternative and added definitions and results done in lecture.

R Problem: p. 187, Problem 6.4.5.

HI Problems:

p. 187, Problem 6.4.2:. Assume $a > b \geq c$. Parts (a) and (b) are just one question: By finding the points (part (b)), one proves they exist (part (a)). *Hints.* Use the calculations in Example 6.4.6. First show every umbilic point must lie on one of the coordinate planes. Your answers will of course be in terms of a, b, and c. The parametrization isn't regular at $(0, 0, \pm c)$, so you must make a separate argument about whether those two points are umbilic points.

p. 188, Problem 6.4.9, modified. Find three asymptotic curves as described in the problem. You do not have to prove these are the only asymptotic curves through the origin. Also find three lines of curvature through the origin. *Hint:* Depending on your approach to the problem, it may be helpful to note the equation for the surface is $z = x^3 - 3xy^2 = x(x+\sqrt{3}y)(x-\sqrt{3}y)$, and that the surface is invariant under rotation by $2\pi/3$ about the z-axis.

Remark. Having more than two asympototic curves and more than two lines of curvature through one point is possible only because it's a planar umbilic point. Can you see why?

Problem S6.2. Let C be the union of the nonnegative coordinate axes in the plane; that is,

$$C = \{(x, y) \in \mathbb{R}^2 : x \geq 0 \text{ and } y = 0\} \cup \{(x, y) \in \mathbb{R}^2 : x = 0 \text{ and } y \geq 0\}.$$

(a) Prove that C is the locus of a C^∞ parametrized curve.

(b) Prove that there is no regular parametrization of C.

Problem S6.3. Suppose S is a regular surface in $\mathbb{R}^3$ and it is contained in some sphere of radius R. Prove that at some point on S, the Gaussian curvature K of S is greater than or equal to $1/R^2$.

(Hint: Prove a slight generalize one of the 442 midterm problems and use that generalized result.)

Problem S6.4. Suppose the locus of a regular curve α lies in the intersection of two oriented regular surfaces S_1 and S_2, and that the angle between the surfaces is constant along the curve. This means that if n_1 and n_2 are the unit normals for S_1 and S_2, respectively, then $n_1 \cdot n_2$ is constant along the curve. Prove that α is principal in S_1 if and only if it is principal in S_2.

See the next page for a mathematical amusement

After working on a problem about lines of curvature, a student in a previous class “happened to find in an attic” (he claimed) the following verses. He described them as “a previously unknown variant of the Rev. Dogdson’s epic, ‘The Hunting of the Snark’.” [Dogdson = Lewis Carroll]

“You may seek it with thimbles, and seek it with care;
    You may hunt it with forks and with hope;
You may threaten its life with a railway-share;
    You may charm it with smiles and with soap.

“But oh, beamish students, beware of the day,
    If your Point be Umbilic! For then
You will softly and suddenly vanish away
    And never be met with again!”

It is this, it is this that oppresses my soul
    When I think of my teacher’s last words:
And my heart is like nothing so much as a bowl
    Brimming over with quivering curds.

Every night - around nine - I draw curvature lines
    In a dreamy delirious state
I parameterize and coordinatize
    In attempts to out-run my fate.

But if ever the Point be umbilic, that day,
    In a moment (of this I am sure),
I shall softly and suddenly vanish away -
    And the notion I cannot endure!