The original problem S2.2 was incorrect, new version below.

Reading: Read Chapter 2 and start Chapter 3.

R Problem:

Problem S2.1. Compute the area of an an ellipse with semi-axes of lengths a and b. (Hint: You should be able to do this by both of the following methods.
(i) Make a change of variables so the ellipse becomes a circle, and apply the change of variables formula for integrals to the integral that gives the area. We will need the change of variables formula in Chapter 6.
(ii) Use Corollary 2.1.4.)

HI Problems:

Problem S2.2. Let $(\mathbb{R}^n, d)$ represent $\mathbb{R}^n$ as a metric space with the distance function d between points defined from the scalar product in the standard way:

$$d(\vec{A}, \vec{B}) = \sqrt{(\vec{A} - \vec{B}) \cdot (\vec{A} - \vec{B})}.$$

Suppose that $F : (\mathbb{R}^n, d) \to (\mathbb{R}^n, d)$ is an isometry that fixes the origin; $d(F(\vec{A}), F(\vec{B})) = d(\vec{A}, \vec{B})$ and $F(\vec{0}) = \vec{0}$. Prove that F preserves the scalar product:

$$F(\vec{A}) \cdot F(\vec{B}) = \vec{A} \cdot \vec{B} \text{ for all } \vec{A}, \vec{B} \in \mathbb{R}^n.$$

Remarks: A map that preserves the scalar product is called an *orthogonal* map. As we noted in class, the converse of Problem 2.2 is easy to prove: an orthogonal map must be an isometry. Problem S2.2 shows that an isometry H of Euclidean space must be of the form $H(\vec{v}) = F(\vec{v}) + \vec{b}$, where F is orthogonal and $\vec{b} = H(\vec{0})$. Later in the course we will see that a smooth map H between two surfaces is an isometry if and only if the derivative $dH(p)$ at each point p is an orthogonal map from the tangent space at p to the tangent space at $H(p)$.

p. 42, #2.1.3(a).

p. 50, #2.2.3.

Problem S2.3. Use the Isoperimetric Inequality to prove that the arclength L of an ellipse with semi-axes of lengths a and b satisfies inequality $L \geq 2\pi\sqrt{ab}$.

p. 58, #2.4.2. *This problem does not require any of the results of Chapter 2. It's placed here in the book because it's needed in the proof of the Four-Vertex Theorem.*

p. 58, #2.4.5.