Assignment 4, due Wednesday, April 29.

Reading: $\S \S 7.2$ and 7.3.
Reading Problem 7.1.15, pp. 235-236, on the Moment of Inertia Tensor may be of interest. This tensor is usually presented as a vector in physics, but as discussed in the last paragraph of the problem, viewing it as a $(0,2)$ tensor makes it possible to work in nonorthogonal coordinates and generalize to higher dimensions.

R Problems: p. 231, \#7.17.
After you do problem S 8.2 below, sketch the vector fields $X_{1}$ and $X_{2}$, and make geometric sense of the covariant derivatives which you compute for S8.2(c).

## HI Problems:

p. 230, \#7.1.1.
p. 244, \#7.2.3 and 7.2.4.

Problem S8.2. Let $S$ be the punctured plane parametrized by polar coordinates: $X(r, \theta)=(r \cos \theta, r \sin \theta)$. For tensor index purposes, consider $r=x^{1}$ and $\theta=x^{2}$.
(a) Write down the coordinate frame $X_{1}\left(=X_{r}\right)$ and $X_{2}\left(=X_{\theta}\right)$, and the Riemannian metric tensor $g_{i j}$ and its inverse $g^{i j}$. Also write down the orthonormal frame $\left\{e_{r}, e_{\theta}\right\}$ you get by normalizing the coordinate frame. (In physics and engineering, coefficients for vectors often are with respect to this the orthonormal frame, but may be described as being the coefficients with respect to the coordinates.)
(b) Find formulas for the gradient of a smooth function $h: S \rightarrow \mathbb{R}$ in terms of the two frames you found in part (a).
(c) Compute the Christoffel symbols for the parametrization $X$, and write down the formulas for the four covariant derivatives $\nabla_{X_{i}} X_{j}$ expressed in terms of the $\left\{X_{1}, X_{2}\right\}$ frame.

