Here are a few notes on Chapter 5. First, some additions to definitions:

Definition 5.1.6: We may also refer to the parametrization X as a *parametrization of* $V \cap S$ or a *parametrization for* S *at* p.

Definition 5.2.9: In point 1, *differentiable* is used to mean C^∞, not just having a (first) derivative. We will call the parametrization X a *regular parametrization.* The inverse map X^{-1} is the *coordinate map* and the functions u and v are the *coordinates* on the *coordinate neighborhood* $V \cap S$.

(Exuberant mathematician to 441 students: a regular parametrization is also called a *smooth embedding.* Note that a smooth embedding is a topological embedding that is also a smooth map, but in addition satisfies the regularity condition (point 3).)

Proposition 5.2.9.5, replacing Proposition 5.2.4 and Corollary 5.2.7.
Suppose X is a regular parametrization of a neighborhood W on a regular surface S, and that $p = X(q) \in W$. Then the tangent space and plane to S at p both exist. The tangent plane at p satisfies equation (5.1) and the tangent space at p is $T_pS = \mathrm{Im}(dX_q)$.

Assignment 6 on the back

Assignment 6, due Wednesday, February 25.

Reading: Read Chapter 5 at least through §3.

R Problem: p. 128, Problem 5.2.6. This should be very easy after doing HI Problem 5.1.1 as modified below.

HI Problems:

p. 84, Problem 3.3.6, modified. We proved the "if" direction in class. Prove the "only if" direction with the additional assumption that $\kappa'(s)$ is never zero. Also find a counterexample to the original statement; that is, find a curve with torsion defined everywhere and satisfying the equation that is not contained in any sphere (and prove this, of course).

p. 113, Problem 5.1.1, modified. Find two parametrizations of the cylinder, one as a surface of revolution and one that is a bijection onto the cylinder. Explain why you cannot restrict the domain of the surface of revolution parametrization to get one that is bijective.

p. 128, Problem 5.2.6. Note that it is not enough to give one parametrization and show it does not satisfy the definition for a coordinate system; you must show that there is at least one point in the surface at which there cannot be any parametrization that satisfies the definition.

p. 128, Problem 5.2.7, modified. Add the hypothesis that the curve is C^∞, and change what you are asked to prove to the following.

1. Prove that if the given conditions hold, then S is a regular surface.
2. Prove that if the given parametrization is a regular parametrization (defined above), then the given conditions hold.
3. Find an example of a surface of revolution for which the given conditions do *not* hold but which is nevertheless a regular surface.

p. 129, Problem 5.2.15, with the following hints and replacement for part (c).

General hint: Sketch the cross section by a plane that contains the z-axis and p and look for similar triangles.

(b) Hint: Just compute $\pi \circ \pi^{-1}$, showing one or two intermediate steps.

(c) Let σ be stereographic projection from the south pole $S = (0, 0, -1)$, so $\sigma : \mathbb{S}^2 - \{S\} \to \mathbb{R}^2$.

1. Find the formula for $\sigma^{-1} \circ \pi$, using geometry. (Do not use the formulas for π or σ to do this.)
2. Find the formulas for σ and σ^{-1}. (Hint: You can do this either by slightly modifying your work in (a) and (b), or by clever use of (c1).)
3. Find the formulas for the coordinate changes $\pi \circ \sigma^{-1}$ and $\sigma \circ \pi^{-1}$. (This can be done using either geometry or the formulas for the individual maps.)

Some notes on Chapter 5 on the front.