Assignment 7, last assignment collected, due Friday, May 29.

Reading: pp. 306-324

R Problems:

Problem S8.5. Prove a compact regular surface in $\mathbb{R}^3$ that is not homeomorphic to a sphere has points where the Gaussian curvature K is positive, is negative, and is zero.

Part (c) of Problem S8.6 below.

HI Problems:

Problem 8.4.4, p. 319. Hint: When $u_2 - u_1 = 2\pi = v_2 - v_1$, your formula for the double integral of K should give a value of zero. (Do you see why?)

Problem 8.4.5, p. 319.

Problem S8.6, a modification of Problem 8.4.6 on p. 319.
Let S be a compact regular surface in $\mathbb{R}^3$.

(a) Assume the Gaussian curvature K is positive everywhere on S. Prove that the Gauss map n is a diffeomorphism.

(b) In problem 8.4.6, p. 319, change the hypothesis that S is homeomorphic to the sphere to the condition that the Gaussian curvature K is positive everywhere on S, and do the modified problem.

(c) **R** problem for class discussion Friday: Sketch or describe a counterexample to the original version of problem 8.4.6, and outline the proof that it is a counterexample. Can you devise a modified definition of area of the image of n that would make the original version true?

Problem 8.4.7, p. 319. Correction: Assume $K \leq 0$ everywhere (not $K \geq 0$).

Problem 8.4.8, p. 320. Remarks and hints: Assuming the curve $P(t)$ is simple means not only that it splits the sphere into only two regions, but that for $t \in I$ the image only goes once around the boundary between the two regions. To simplify the calculations, you may assume that one of the curves (α or P) is unit speed (but you cannot assume *both* are unit speed). Late in the calculations, you should find it useful to recall that for f and g functions of t,

$$\frac{d}{dt}\arctan\left(\frac{f}{g}\right) = \frac{f'g - fg'}{f^2 + g^2}.$$