Corrections and clarifications made to this update: This is Assignment 7, not 6. (Doh!)
In Problem 5.4.6, the given conditions imply F is a diffeomorphism.
In Problem S6.1(a), c is a constant.

Assignment 7, due Friday, March 6.

Reading: Finish Chapter 5, read §6.2, and start on §6.1.

R Problems:

p. 137, Problem 5.4.1.

p. 143, Problem 5.5.2.

p. 165, Problem 6.2.2.

HI Problems:

p. 130, Problem 5.2.16(a).

p. 134, Problem 5.3.5.

p. 138, Problem 5.4.6. Note: Proposition 5.4.2 implicitly assumes that F is a diffeomorphism; so in particular you may use the equations on p. 133. (We are given that $Y = X \circ F$ is a regular parametrization, so Y is injective and dY has rank 2 at every point. This implies the same conditions hold for F.)

p. 165, Problem 6.2.1(bcd). Be sure to specify your choice of orientation for each surface. Your description of the image should be specific; for example, something like "the open lower hemisphere" or "every point on the sphere except ..." or "the intersection of the sphere with the plane ...".

p. 166, Problem 6.2.4, with the following modifications.

(a) Assume that the curve parametrization $\vec{\alpha}$ in 5.2.16(a) is regular. Prove that for your parametrization X of the cone, every point in the cone except the vertex is a regular value of X.

The reference to a "one-sheeted cone" means to assume $\vec{\alpha}$ is injective. You should actually make a stronger assumption, that your parametrization X is a regular parametrization except at the vertex, so the cone minus its vertex is a regular surface. (This can be proved if $\vec{\alpha}$ is a homeomorphism to its image.)

(b) Show that the image of the Gauss map is a curve by finding a parametrization of the image as curve in terms of $\vec{\alpha}$.

Problem S6.1. Orient the surface $S = \{(x, y, z) \in \mathbb{R}^3 \,:\, z = xy\}$ by the upward pointing normal. (That is, the normal has positive z-component.)

(a) Find the image under the Gauss map $n : S \to \mathbb{S}^2$ of the curve $\vec{\alpha}_c(t) = (t, c, ct)$, where $c \in \mathbb{R}$ is constant. Hint: You should find that it is part of a great circle; which great circle, and what part of it?

(b) Describe the region of the unit sphere covered by the image of the Gauss map on S. Be specific, as indicated for Problem 6.2.1 above.