On the first quiz, you were expected to show almost all the details of reasoning for separating variables and many of the details for solving the separated problems. Starting with the homework due after the quiz, you may omit a lot of these details. When separating variables, you may immediately write down the separated equations. If you are solving a problem you have solved before, for instance,

$$X'' + AX = 0, \qquad X(0) = 0 = X(\ell),$$

you may immediately write down the family of independent solutions. You should include the corresponding value of the separation constant/eigenvalue ($A$ in the problem above) and the range for any index such as $n$. If you find you need to look up the solution, for instance in §1.3, do say where you found it; then it's easier for us to see your thinking if you make an error. But if you have learned it by heart, you may just state it.

*Of course if you are at all unsure what the correct answers are, you should show any steps you need to do. In particular you might be cautious in this way if any of the equations are different from cases you've previously dealt with.*

Here is an example: a solution for §1.3, #6 (p. 17) under these new rules. (When you did this for homework two weeks ago, more detail was required.)

**The problem** was to find an infinite family of solutions to

$$u_{xx} + u_{yy} + u_{zz} = 0,\ u(0, y, z) = u(1, y, z) = u_y(x, 0, z) = u_y(x, 1, z) = 0.$$

*Solution.* Optional: write down

$$\frac{X''}{X} + \frac{Y''}{Y} + \frac{Z''}{Z} = 0.$$

(I'd find this helpful to make sure I get the relation among the separation constants correct.)

Separated problems:

$$\begin{array}{rll} \text{For } X: & X'' + AX = 0, & X(0) = 0 = X(1); \\ \text{for } Y: & Y'' + BY = 0, & Y'(0) = 0 = Y'(1); \\ \text{and for } Z: & Z'' - (A + B)Z = 0. & \end{array}$$

(You could write third equation as $Z'' + CZ = 0$, but then must also say $A + B + C = 0$.)

Solutions for $X$:

$$X(x) = \sin(n\pi x), A = (n\pi)^2, \text{ for } n \in \mathbb{N} \text{ (or } n = 1, 2, 3, ...).$$

Solutions for $Y$:

$$Y(y) = 1 \text{ for } B = 0,\ Y(y) = \cos(m\pi y), B = (m\pi)^2, \text{ for } m \in \mathbb{N} \text{ (or } m = 1, 2, 3, ...).$$

Remarks, not part of solution: (i) Note the use of a different index, $m$, because we will have to combine the solutions for $X$ and $Y$. The integer in the $X$ function in general will not be the same as the one in the $Y$ function. (ii) You can use the book's trick of including the constant solution in the cosine case by including $m = 0$, if you like. If you do, be careful this doesn't lead you to think there is always a constant solution if there are cosine solutions; consider the quiz problem!

Solutions for $Z$: For $C = -(A + B) = -[(n\pi)^2 + (m\pi)^2]$, we solve $Z'' + CZ = 0$ to get two linearly independent solutions

$$Z(z) = \exp(\sqrt{n^2 + m^2}\pi z) \text{ or } Z(z) = \exp(-\sqrt{n^2 + m^2}\pi z).$$

Here $n$ can be any positive integer and $m$ can be any nonnegative integer.

Thus we get a family of independent solutions

$$\begin{aligned}
u^+_{n0}(x, y, z) &= \sin(n\pi x) \exp(n\pi z), \\
u^-_{n0}(x, y, z) &= \sin(n\pi x) \exp(-n\pi z), \\
u^+_{nm}(x, y, z) &= \sin(n\pi x) \cos(m\pi y) \exp(\sqrt{n^2 + m^2}\pi z), \\
u^-_{nm}(x, y, z) &= \sin(n\pi x) \cos(m\pi y) \exp(-\sqrt{n^2 + m^2}\pi z),
\end{aligned}$$

for $n, m \in \mathbb{N}$.

The book's final answer has $n$ and $m$ switched compared with this one, and by notational tricks includes all four cases in one formula. I like separating the solutions to see more clearly what they are, but this is a matter of personal taste.