Problem E. Do p. 67, #3a,b,c, and

d. Suppose for some reason you didn't realize the space in question is three dimensional, so you didn't bother to find a third vector for your basis. (This sounds silly. But when we work in infinite dimensions, it's not so easy to be sure we have "enough" vectors in a basis.) You can't express the vector $a = (1, 2, 3i)$ as a linear combination of y_1 and y_2; but what's the "best" you can do? Well, you should minimize the difference between a and the linear combination. In particular it is possible to choose the coeffients so that this difference is orthogonal to y_1 and y_2. Find the coefficients that accomplish this. Compare to your answer in part c. Interpret what you are doing geometrically to see why this choice will minimize the difference. (As the book suggests, visualize three real dimensions even though you are working over the complex numbers. This problem covers the same ideas as #5 on p. 68.)

Problem F. Let $f(x) = \sin(x)$, $g(x) = \sin(2x)$, and $h(x) = \cos(2x)$. Answer the following WITHOUT doing any integration. (It may be helpful to draw graphs of the three functions; or you can use what you know about Fourier series on an interval $[0, \ell]$.)

(i) Which of f, g, and h are orthogonal to $\cos(x)$ in $L^2(0, \pi)$?

(ii) Which of f, g, and h are orthogonal to cos(x) in $L^2(0, \pi/2)$?

(iii) Why can't you answer the question "Is $\cos(x)$ orthogonal to $\sin(x)$?" (Something is missing in the question; what is it and why is it important?)

Problem G. (This is a modified version of §3.3, pp. 80-81, #9 & 11.) In b & c below, be careful when using results from §2.3: f is smooth but f' is only known to be continuous (not necessarily smooth or piecewise smooth). Also for b & c, explain why part a applies. In b, c, and d, indicate when you are using the fact that f is real-valued.

a. State and prove a version of #9 for a complete orthogonal (but not necessarily normal) set $\{\psi_n\}$ for $L^2(a, b)$. (Hint: Thinking about #2, p. 67, may be helpful.)

b. Do part (a) of #11 using the real form of the Fourier series ((2.1), p. 19).

c. Do part (a) of #11 using the complex form of the Fourier series ((2.2), p. 19).

d. Do part (b) of #11.

Problem H. Consider the graphs on p. 27. Without looking at the series in the table on p. 26, use symmetry to answer the following questions.

(a) Is a_0 zero, positive, or negative?

(b) Are the rest of the a_n zero? What's the sign of the first nonzero one (if any)? Can you make a general statement about the signs of the nonzero coefficients? (Hint: sometimes it helps to subtract off a_0.)

(c) What about the b_n? (Same questions as for a_n.)