Problem A. You will be solving the ODE $f''(x) + bf(x) = 0$ repeatedly in this course, and it will be useful to express the general solution in various forms. Some are given on p. 13. In this problem, you will find several other forms we will need, some soon and some later.

Prove the following. You may use Theorem 1.1 on p. 13 and the paragraph that follows it (ending with the statements in italics and numbered (i) and (ii)).

1. If $b = 0$, then the general solution is $f(x) = B_1 + B_2 x$.

2. If $b = \alpha^2 > 0$ (which implies b is real), then the general solution can be written in the following three ways:

$$f(x) = C_1 e^{i\alpha x} + C_2 e^{-i\alpha x} = C_3 \cos(\alpha x) + C_4 \sin(\alpha x) = C_5 \sin(\alpha x + C_6).$$

   (Hint: If you get stuck, try working backwards from what you are trying to get; then check that all your steps are reversible.)
   Give equations relating (C_1, C_2) to (C_3, C_4) and relating (C_3, C_4) to (C_5, C_6).

3. If $b = -\alpha^2 < 0$ (which again implies b is real), then the general solution can be written in these two ways:

$$f(x) = D_1 e^{\alpha x} + D_2 e^{-\alpha x} = D_3 \cosh(\alpha x) + D_4 \sinh(\alpha x).$$

   Give equations relating (D_1, D_2) to (D_3, D_4). Given a constant $L > 0$, the general solution can also be written in the following ways,

$$f(x) = D_5 \cosh(\alpha x) + D_6 \cosh(\alpha[L - x]) = D_7 \sinh(\alpha x) + D_8 \sinh(\alpha[L - x]).$$

   Give equations relating (D_1, D_2) to (D_5, D_6) and (D_1, D_2) to (D_7, D_8).

Problem B. For formula #6 in the table, sketch *by hand* the graphs of partial sums S_N for $N = 1$, 3, and 5, as discussed in class. (Notation: S_N as in (2.11) on p. 33; that is, the highest frequency term in S_3 is the $\sin(3\theta)$ term.)

If you have the skills to do this easily by computer, I encourage you to graph partial sums for several higher values of N.

Problem C. (i) Do p. 48, #7.
(ii) Two rooms have been held at $20^o C$ for so long that the thick wall (6π cm.) between them is also at $20^o C$ throughout. Suddenly the air in both rooms is replaced by air at $0^o C$, and is held at that temperature. Assume that the interface between air and wall is perfectly conducting. (This is bad physics; we'll replace this with a more realistic assumption in a couple of weeks.) Find the temperature in the wall as a function of position and time. Your solution will be in terms of the "thermal diffusivity" k. Hint: You should find part (i) useful.

Problem D. (i) Do p. 48, #9.
(ii) A thin, homogeneous cylinder 2 feet long at a uniform temperature of 10 degrees is placed end to end with a cylinder of the same size, shape, and material at a uniform temperature of -10 degrees. The interface is perfectly conducting and all other sides are perfectly insulated. Find the temperature as a function of position and time. Your solution will be in terms of the "thermal diffusivity" k. Hint: you should find part (i) useful.